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Vol. 248 DECEMBER, 1949 No. 6

GIANT MOLECULES IN SOLUTION *

BY

THE SVEDBERG [1]

In a primeval sea, life probably originated from further development of giant molecules in solution. How these were formed we do not know—as a possible cause we might assume some chance combination of amino acids. In the solid state well-defined, closed molecules are rare and we are, therefore, led to believe that such entities as protein molecules were first built up in a solution. At present we find them only as synthesis products in living beings. The viruses occupy an intermediate position between dead and living matter, between protein molecules and bacteria and they are known to exist in a liquid or semi-solid state.

Such giant molecules as we know from the solid, crystalline state and which are incapable of directly going into solution without the action of chemical agents are not so well-defined. They mostly represent the repetition of a basic unit in various degrees of polymerization.

It was through physico-chemical studies of protein solutions that the existence of well-defined giant molecules was found. The colloid character of proteins was recognized at an early date—as a matter of fact the term "colloid" was coined from observations on a protein, gelatin—but the monodispersity or paucidispersity of these substances could not be proved by means of the methods in use at that time.

Convection-free sedimentation in the ultracentrifuge combined with refined diffusion measurements gave the surprising result that most of the proteins in solution and some of those in the condensed state were built up of well-defined molecular units. An enthusiastic team of young research men—rather international in character—carried out the first observations of this kind in Upsala (the American Nichols, the Austra-

* Talk presented at the Medal Day Meeting at The Franklin Institute, October 19, 1949. (Read by Dr. J. Burton Nichols, in the absence of the author.)

[1] Director, The Gustaf Werner Institute for Nuclear Chemistry, Upsala, Sweden.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

lian Lewis, the Swede Fahraeus, the Romanian Chirnoaga). We had happened to hit upon representatives of quite a few different classes of proteins for our first experiments, ranging in molecular weights from about 40,000 (egg albumin), about 70,000 (hemoglobin), about 150,000 (phycocyan), about 300,000 (phycoerythrin) to about 9,000,000 (hemocyanin). A rapid development followed, especially in America, England and Sweden, both with regard to experimental tools and the investigation of substances. At present sedimentation constants can be measured with an accuracy better than 1 per cent.

In principle the sedimentation experiments are carried out in the following way. A small sample of the solution to be studied is enclosed between quartz windows in a sector-shaped cell, put into a suitable rotor (in Upsala we use steel) and spun at high speed in low pressure (the Upsala procedure) or in a vacuum. Observations of the sedimenting boundary in the solution are carried out by means of refractive index or light absorption measurements. From the data thus obtained the sedimentation constant is computed and, by combination with the diffusion constant, the molecular weight can be found.

Among the results on protein molecules, existing spontaneously in solution, may be mentioned the following ones. The respiratory proteins are exceedingly well-defined. Their molecules possess one or a few prostetic groups, often containing a metal atom (iron, copper). The lowest molecular weights observed are those of cytochrom (12,000) and myoglobin (17,000). The red pigment of the nitrogen bacteria is 34,000 and the hemoglobin of the vertebrates 68,000 with the exception of the lowest class, the Cyclostomata, the members of which have a blood pigment of molecular weight 17,000. The copper-containing hemocyanines of the Gastropodes and the Crustaceans present a number of quite interesting features. By raising or lowering the pH or, in certain cases, simply by the addition of salt, the molecule often dissociates into halves and eighths. The hemocyanin of the vineyard snail contains two kinds of molecules in the proportion 3:1. Both types are dissociated by certain pH-changes but the more abundant type also by salts. As an other example of reversible pH-dissociation one could mention the phycocyan, the blue pigment of the algae, the molecule of which splits into halves already at a pH of 6.8.

The serum proteins represent a very important but also very complex group. Especially the globulins are numerous. In pathological sera new constituents often appear, or the proportions of the normal ones are changed. In the serum of the fetus and the new-born of certain animals (ox, horse, sheep) a new protein, named the fetuin, was recently demonstrated. With age this constituent disappears. A comparative study of the different vertebrate classes shows that the serum of the birds closely resembles that of the mammals. The reptiles, the amphibians and the fishes possess sera of similar, though slightly different,

constitution, while the serum of the lowest class of the vertebrates, the Cyclostomata, is entirely different from all other vertebrate sera.

Among the proteins occurring naturally in the condensed state, but extractable by salt solutions, may be mentioned the vegetable globulins and the muscle proteins. The former ones are quite well-defined and often characteristic of their respective families; the latter ones, which (in contradistinction to most of the proteins mentioned above) have thread-like molecules, are not quite monodisperse. It seems to be the rule that globular macromolecules represent closed entities, while thread-like molecules can vary in size within certain limits, probably because of the ease with which sub-units may be added or subtracted.

The polysaccharides which play such an important part within the vegetable kingdom as support or reserve substances are mostly built up of thread-like macromolecules (cellulose, pectines). Only a few polysaccharides, such as the arabogalactane of the larch tree, has globular molecules. Also in the case of the polysaccharides the substances possessing thread-like molecules (such as cellulose) are more or less polydisperse, while those composed of globular molecules are monodisperse. The polydispersity of cellulose is, however, less pronounced than would be the case if its molecules had been formed by chance polymerization of glucose units. This shows that the formation of cellulose in the plants follows ways quite different from those which we are used to observe when high polymers are synthesized in vitro. The distribution curve for the molecular weights of native cellulose, as recently determined in Upsala, possesses several maxima, indicating the accumulation of certain molecular sizes in distinct groups.

The cellulose as it occurs in the wood is closely bound to lignin and it takes a certain chemical treatment to liberate it. If this is performed with care the product appears to be equally well-defined as the much less firmly bound cellulose of the cotton-seed hairs. Both in the case of wood cellulose and cotton cellulose the distribution curve shows maxima at a degree of polymerization of about 100, 200 and 300 glucose units. It is now of considerable interest that it has been possible, quite recently, to prepare by partial hydrolysis of native cellulose a sort of cellulose colloid built up of fairly equally sized rods possessing a length corresponding roughly to 100 glucose units as determined in the electron microscope. Electron diffraction studies on these cellulose rods have shown that they possess a very regular crystal lattice of the same general type but better defined than, according to previous X-ray studies, the cellulose fibers. Apparently the micelles as they occur in the fibers are partially hidden by amorphous material and thus less accessible to X-ray studies than in the isolated state. All this tends to convince us that in these rods we are confronted with the real elementary cellulose particles. These consist of bundles of parallel molecules of the type observed by means of the ultracentrifuge in cellulose solutions.

Ultracentrifugal and diffusion studies on synthetic polymers such as polymethacrylates and polychloroprene have demonstrated the chance distribution of their molecular weights. It has further been possible to show, in the case of chloroprene, that bulk polymerization produces chain-like, free-draining molecules, while in emulsion polymerization the molecules produced resemble matted coils.

The importance of high-molecular substances, both natural and synthetic, in biochemistry, biology, medicine and technology is becoming increasingly evident. When I started work along the lines mentioned above 25 years ago, interest in these things was rather limited. Largely through the encouragement I received in the United States during a stay at the University of Wisconsin in 1923 and later on through efficient financial help from the Rockefeller Foundation and through the enthusiastic collaboration with numerous colleagues and students from many countries, but foremost from the United States, I was led to continue my work on high molecular substances during many years. It is for me a great satisfaction that my endeavors have now been rewarded by this high distinction conferred upon me, The Franklin Medal.

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1949

RESPONSE OF AN ELASTICALLY NON-LINEAR SYSTEM TO TRANSIENT DISTURBANCES

BY

RUNE L. EVALDSON,[1] ROBERT S. AYRE,[1] AND LYDIK S. JACOBSEN[1]

ABSTRACT

The problem relates to an undamped, elastically non-linear, single-degree-of-freedom system subjected to three forms of ground motion pulses (rectangular, cosine and "skewed" cosine); each form consists of a single pulse, the duration of which has been varied over a wide range. The maximum relative displacements (maximum distortions) have been plotted in dimensionless form as functions of the pulse durations. Under some circumstances the non-linearity results in reduced maximum distortion, but in other cases it acts to increase it. Simple quantitative conclusions cannot be drawn. Analytical, graphical and experimental methods were employed; the graphical method has been presented in detail.

INTRODUCTION

The problem of a single-degree-of-freedom mechanical vibratory system subjected to a transient disturbance is found in many fields of engineering. When the restoring elements are linear the solution is straight forward but when they are non-linear it is often necessary to use experimental, graphical and approximate analytical methods. While the effects of *steady-state* disturbances on the non-linear case are fairly well known (1, 2, 3, 4)[2] the *transient state* has not received much attention (6, 7).

The investigation[3] consists of an experimental and graphical determination of the response to transient *ground motions*. The non-linearity of the restoring element is symmetrical with respect to the instantaneous position of the moving ground and involves an abrupt but constant increase in stiffness at a given relative displacement between the ground and the mass. The relative displacement at which the increase in stiffness becomes effective and the ratio of the two constant stiffnesses have been varied over wide ranges. The study has been limited to the case of zero damping in order that primary emphasis could be placed on the effect of the non-linearity.

The ground motions were of various single-pulse types, including the rectangular, the cosine and the "skewed" cosine shapes. The periods or durations of the pulses were varied widely. For comparative

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] Sponsored by ONR at the Vibration Laboratory, Stanford University.

purposes the theoretical responses of the limiting linear systems to the same pulses are given.

The experimental portion of the program made use of a translational mechanical vibrating system which was subjected to the pulses by means of a cam-driven mechanical linkage. The graphical work employed a stepped approximation to the actual ground motion and a method (5, 6) which is related to the phase-plane method of non-linear mechanics.

THE VIBRATORY SYSTEM

The idealized vibratory system is shown schematically in Fig. 1, and the restoring force-relative displacement function in Fig. 2.

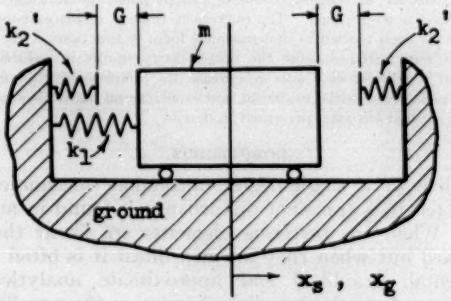

FIG. 1. The idealized vibratory system.

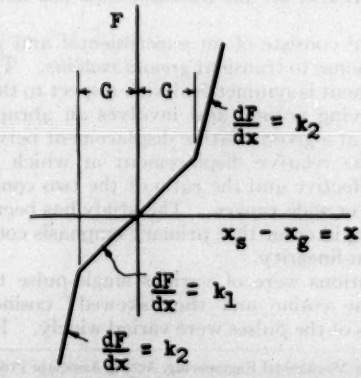

FIG. 2. Restoring force—relative displacement function.

The differential equation of motion is

$$m\ddot{x}_s + Kx_s = Kx_g, \tag{1}$$

where the notation is as follows:

x_s = absolute displacement of the mass of the system,
x_g = absolute displacement of the ground,
X_g = maximum value of x_g,
t = time, measured from start of the ground pulse,
t_g = period or duration of the ground pulse,
t_m = time of occurrence of X_g,
σ = degree of "skewness," defined as t_m/t_g,
ω = ground pulse frequency $= 2\pi/t_g$,
$x = x_s - x_g$ = relative displacement of mass and ground (distortion),
X = absolute value of the maximum of the maxima of x for a particular system when subjected to a particular ground pulse,
X_m = the maximum X for a particular system when subjected to pulses of like form and amplitude but where t_g is varied,
m = mass of the system,
G = a distance (spring gap),
k_1 = spring constant of a restoring element which is in action at all values of x,
k_2' = spring constant of a restoring element ("snubbing" spring) which acts only when $|x| > G$,
$k_2 = k_1 + k_2'$,
T = natural period of a linear system,
T_1, T_2 = natural periods of the limiting linear systems of a bi-linear family,
τ = degree of bi-linearity, defined as $\sqrt{\frac{k_2}{k_1}}$, that is, $\frac{T_1}{T_2}$,
F = restoring force, and
$K = \frac{F}{x}$ = instantaneous spring constant.

The natural period of the system shown in Fig. 1 is in general a function of displacement. However, the system will be linear in the two limiting cases, namely:

1. When $\frac{X}{G} \lessapprox 1$, $K = k_1$ and $T_1 = 2\pi\sqrt{\frac{m}{k_1}}$. (2a)

2. When $\frac{X}{G} \to \infty$, $K = k_2$ and $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$. (2b)

Under each of conditions (2a) and (2b) the system is linear for all values of x. These limiting systems provide points of departure in the study of the non-linear family lying between them.

Since the restoring force-relative displacement curve is composed of three straight lines with two slopes, k_1 and k_2, the system will be termed

bi-linear. It approximates physical systems incorporating snubbing- or stop-springs. Furthermore, spring characteristics of continuous non-linearity may be reduced without excessive error to sequences of linear elements, of which the bi-linear characteristic is the simplest.

The degree of bi-linearity will be denoted by τ, with the definition:

$$\tau = \frac{T_1}{T_2} = \sqrt{\frac{k_2}{k_1}}. \tag{2c}$$

THE GROUND MOTIONS

The system was subjected to three forms of single pulse ground motion:

Form 1. Rectangular Pulse, (Fig. (3a)):

$$x_g = X_g \quad \text{for} \quad 0 \leqq t \leqq \frac{t_g}{2}, \tag{3}$$

$$x_g = 0 \quad \text{for} \quad \frac{t_g}{2} \leqq t \leqq t_g.$$

Form 2. "Skewed" Cosine Pulse, (Fig. (3b)):

$$x_g = X_g \frac{e^{(2\pi\sigma - \omega t)\cot\pi\sigma}}{1 - \cos 2\pi\sigma}(1 - \cos \omega t) \tag{4}$$

for $0 < \sigma < \frac{1}{2}$.

Form 3. Cosine Pulse, (Fig. (3c)):

$$x_g = \tfrac{1}{2}X_g(1 - \cos \omega t). \tag{5}$$

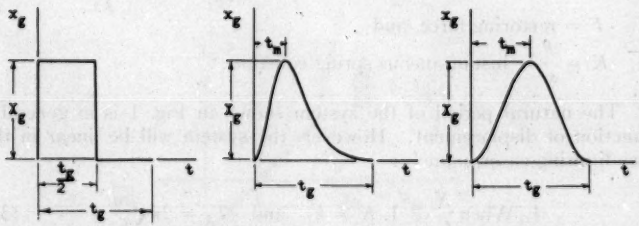

FIG. 3. The ground motion pulses (a) Rectangular pulse (form 1). (b) Skewed cosine pulse; $0 < \sigma < \frac{1}{2}$ (form 2). (c) Cosine pulse; $\sigma = \frac{1}{2}$ (form 3).

Form 2 represents a family of motions obtainable by variation of σ, the degree of "skewness." In the range $0 < \sigma < \frac{1}{2}$ the family provides examples of the common type of ground motion which progresses to a maximum and then recedes less rapidly to its original position. The portion of the family obtained by letting $\frac{1}{2} < \sigma < 1$ is of less interest. Form 3 is a special case of Form 2 where $\sigma = \frac{1}{2}$.

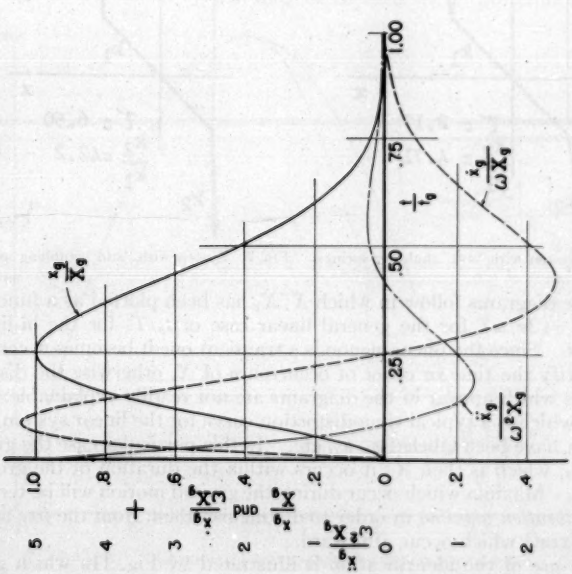

FIG. 4. Displacement, velocity and acceleration of ground; skewed cosine ground pulse; $\sigma = \frac{1}{4}$.

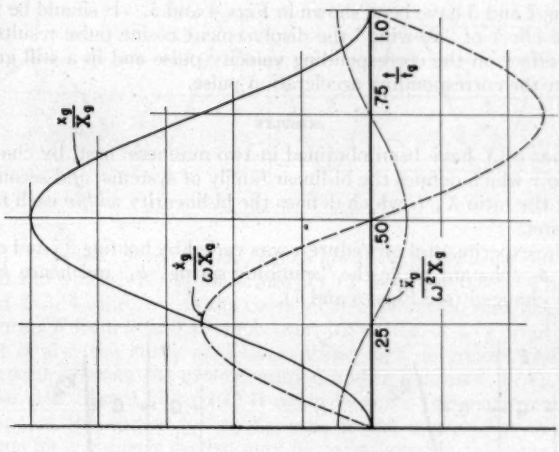

FIG. 5. Displacement, velocity and acceleration of ground; cosine ground pulse; $\sigma = \frac{1}{4}$.

The displacement, velocity and acceleration of the ground for pulses of Forms 2 and 3 have been shown in Figs. 4 and 5. It should be noted that the effect of "skewing" the displacement cosine pulse results in a greater effect on the corresponding velocity pulse and in a still greater effect on the corresponding acceleration pulse.

RESULTS

Values of X have been obtained in two manners: first, by changing the ratio τ which defines the bi-linear family of systems; and second, by varying the ratio X_o/G which defines the bi-linearity *within* each family of systems.

In the experimental procedure τ was varied by holding T_1 and consequently k_1 constant while the "snubbing spring" k_2' and hence k_2 and T_2 were changed (see Figs. 6 and 7).

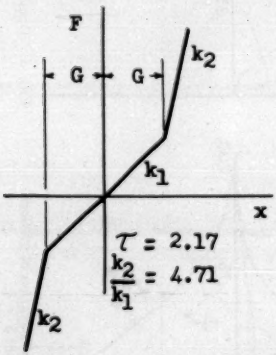

FIG. 6. System with "soft" snubbing springs.

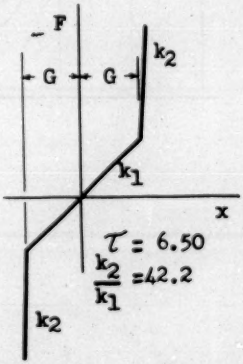

FIG. 7. System with "stiff" snubbing springs.

Nine diagrams follow in which X/X_o has been plotted as a function of $t_o/T = 2\pi/\omega T$ for the general linear case or t_o/T_1 for the bi-linear systems. Since the phenomenon is a transient one it becomes necessary to identify the time or order of occurrence of X, otherwise the discontinuities which appear in the diagrams are not readily explainable. In Fig. 8, which is a typical time-distortion curve for the linear system, the maxima have been labeled x_1, x_2, etc. In this particular case the greatest is x_3, which is then X; it occurs within the duration of the ground motion. Maxima which occur during the ground motion will be termed *forced vibration maxima* in order to distinguish them from the *free vibration maxima* which occur afterward.

The use of the identification is illustrated by Fig. 11a which gives the response of the linear system to the cosine pulse. The curve labeled

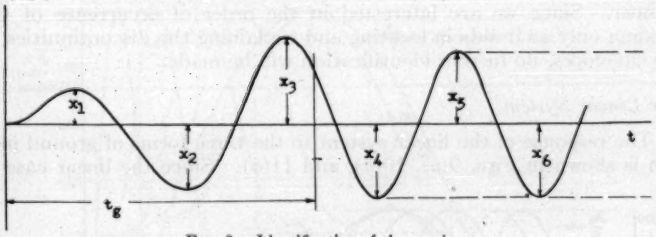

FIG. 8. Identification of the maxima.

1, 1, 1, $\cdots$ is the curve of x_1/X_g which always occurs during the ground motion and which is X/X_g for values of t_g/T from 0 to 0.30. The curve labeled 2, 3, 4, 5, 6, $\cdots$ is the curve of free vibration maxima, x_2/X_g, etc., for t_g/T from 0 to 0.48; moreover, for values of t_g/T from 0.30 to 0.48 it is also the curve of X/X_g. When t_g/T is greater than 0.48, x_2/X_g occurs during the ground motion and is less than X/X_g, except that for t_g/T from 1.67 to 2.42 it again becomes the maximum. The envelope of the curves for x_1/X_g, etc., is the curve of X/X_g. The envelope for a bi-linear system may be constructed in the same general

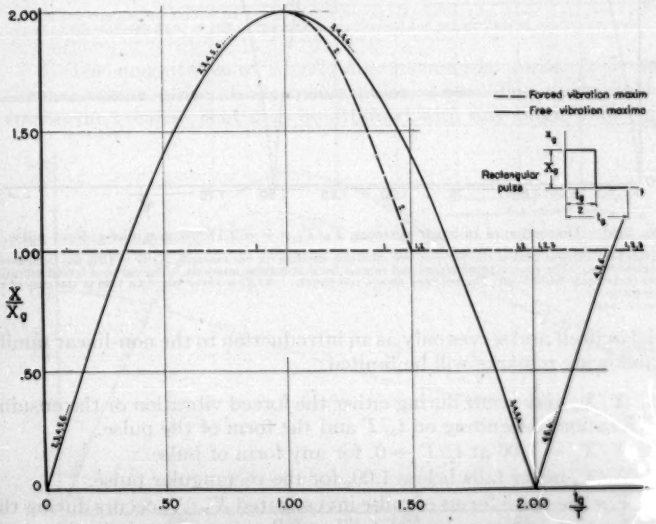

FIG. 9(*a*). Distortion of linear system; rectangular ground pulse.

The numbers on the curves refer to consecutive maxima (see Fig. 8). The peak of the envelope is symmetrical and repeats itself at regular intervals; X_m/X_g occurs at $t_g/T = 1, 3, 5$, etc.

manner. Since we are interested in the order of occurrence of the maxima only as it aids in locating and explaining the discontinuities in the envelopes, no further identification will be made.

The Linear System

The response of the linear system to the three forms of ground motion is shown in Figs. 9(*a*), 10(*a*), and 11(*a*). Since the linear case is

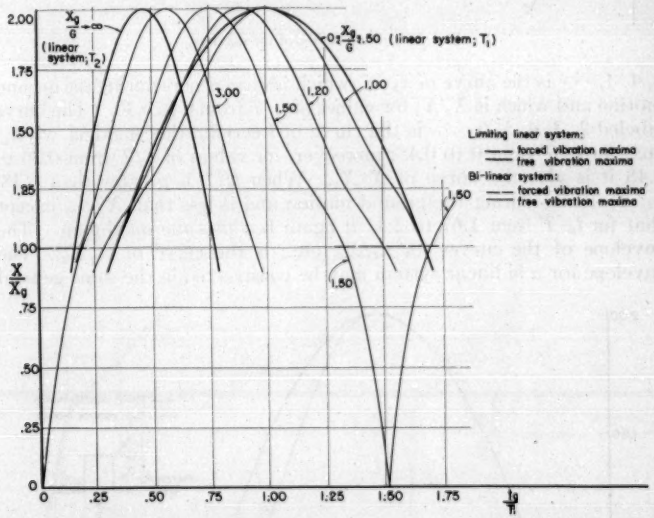

FIG. 9(*b*). Distortion of bi-linear systems, $T_1/T_2 = \tau = 2.17$; rectangular ground pulse.

Note that the first peak of the curve of free vibration maxima for one example, $X_g/G = 1.50$, has been shown in complete form. The peak is symmetrical and repeats itself at regular intervals. The curve of forced vibration maxima (horizontal dashed line) has not been shown completely. X/X_g is never less than 1.00 in the case of the rectangular pulse.

trivial in itself and serves only as an introduction to the non-linear family the following remarks will be limited:

1. X/X_g may occur during either the forced vibration or the ensuing free vibration, depending on t_g/T and the form of the pulse.
2. $X/X_g = 1.00$ at $t_g/T \to 0$, for any form of pulse.
3. X/X_g never falls below 1.00, for the rectangular pulse.
4. For the three forms of pulse investigated X_m/X_g occurs during the free vibration and at values of t_g/T as follows: rectangular pulse, 1.00, 3.00, 5.00, etc.; quarter skewed cosine pulse, 1.05; cosine pulse, 0.84.

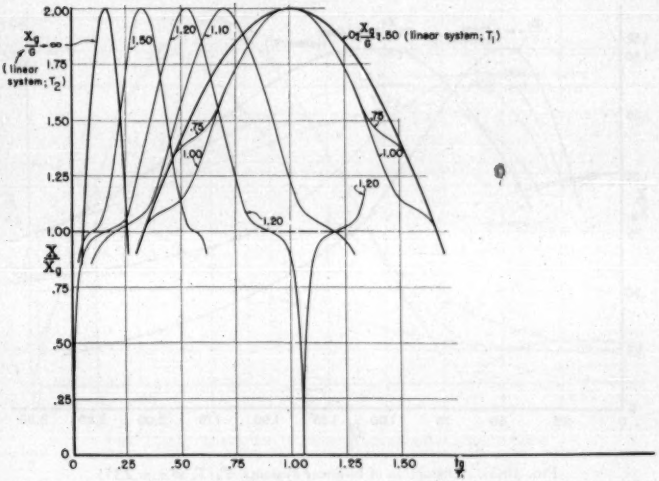

FIG. 9(c). Distortion of bi-linear systems, $T_1/T_2 = \tau = 6.50$; rectangular ground pulse.
In this case the first peak of the curve of free vibration maxima has been shown in complete form for the example, $X_s/G = 1.20$. The peak is symmetrical and repeats itself at regular intervals.

5. The magnitudes of X_m/X_g are: rectangular pulse, 2.00; quarter skewed cosine pulse, 1.52; cosine pulse, 1.64. The value 2.00 is of course the greatest that may be attained with any single pulse.

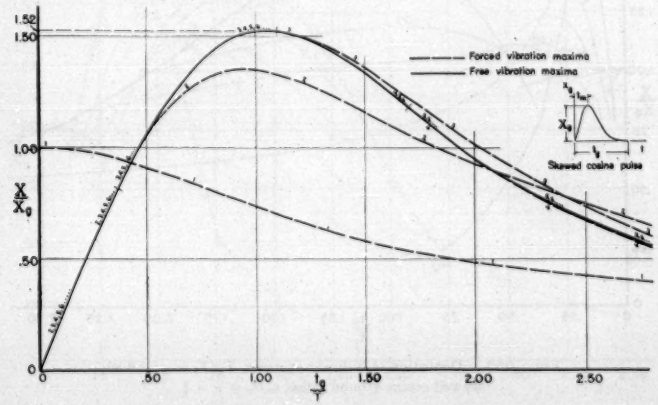

FIG. 10(a). Distortion of linear system; skewed cosine ground pulse, $t_m/t_g = \sigma = \frac{1}{4}$.

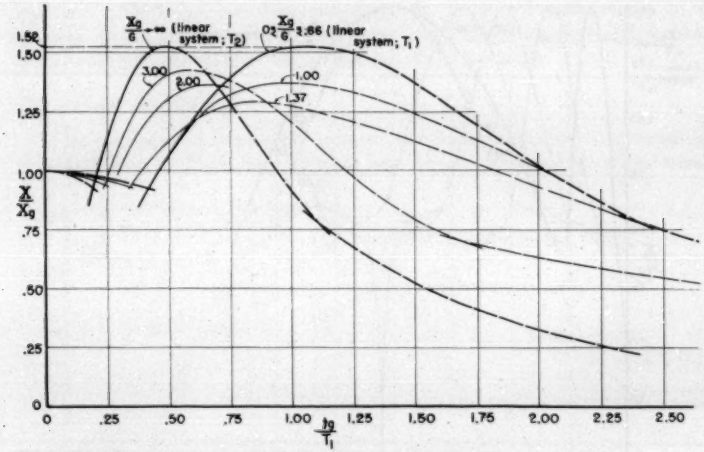

FIG. 10(*b*). Distortion of bi-linear systems, $T_1/T_2 = \tau = 2.17$; skewed cosine ground pulse, $t_m/t_g = \sigma = \frac{1}{4}$.

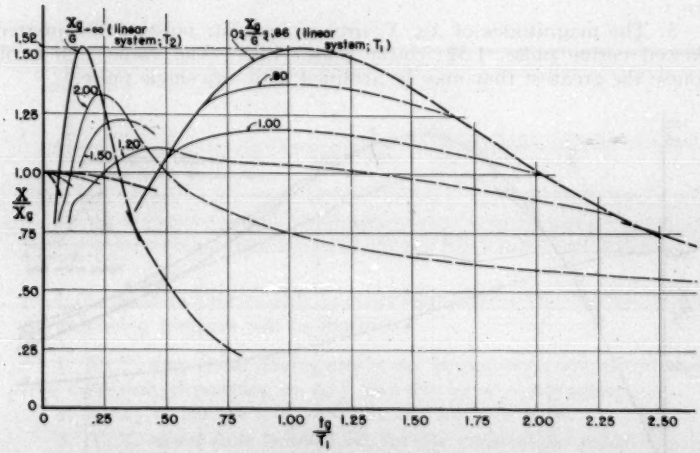

FIG. 10(*c*). Distortion of bi-linear systems, $T_1/T_2 = \tau = 6.50$; skewed cosine ground pulse, $t_m/t_g = \sigma = \frac{1}{4}$.

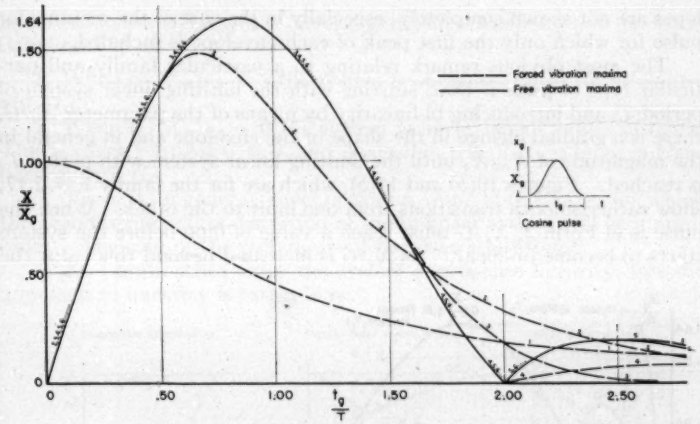

FIG. 11(*a*). Distortion of linear system; cosine ground pulse.

The Families of Bi-Linear Systems

The envelopes of maximum distortion are shown in Figs. 9(*b*), 9(*c*), 10(*b*), 10(*c*), 11(*b*), and 11(*c*). For reasons of clarity some of the enve-

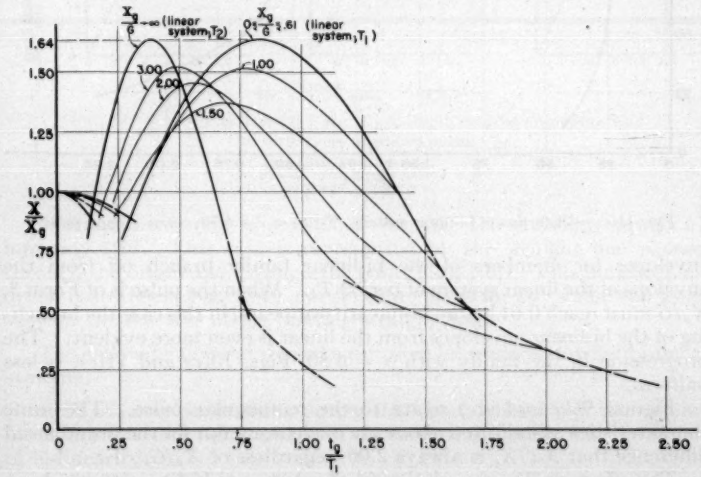

FIG. 11(*b*). Distortion of bi-linear systems, $T_1/T_2 = \tau = 2.17$; cosine ground pulse.

lopes are not shown completely, especially in the case of the rectangular pulse for which only the first peak of each envelope is included.

The most obvious remark relating to a particular family and particular type of pulse is that, starting with the limiting linear system of period T_1 and introducing bi-linearity by means of the parameter X_g/G, there is a gradual change in the shape of the envelope and in general in the magnitude of X_m/X_g until the limiting linear system with period T_2 is reached. Figures 10(*b*) and 11(*b*), which are for the family $\tau = 2.17$, show rather smooth transitions from one limit to the other. When the pulse is of Form 2, X_g/G must reach a value of 0.66 before the system starts to become bi-linear. As X_g/G is increased beyond this value the

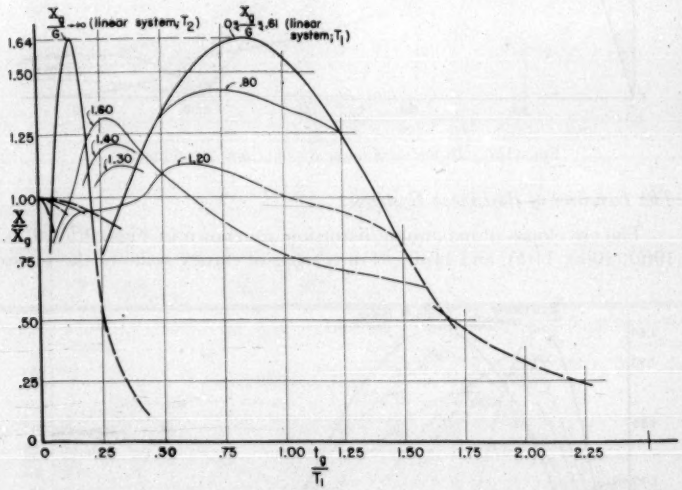

FIG. 11(*c*). Distortion of bi-linear systems, $T_1/T_2 = \tau = 6.50$; cosine ground pulse.

envelopes for members of the bi-linear family branch off from the envelope of the linear system of period T_1. When the pulse is of Form 3, X_g/G must reach 0.61 before bi-linearity appears; in this case the branching of the bi-linear envelopes from the linear is even more evident. The progression in the family with $\tau = 6.50$, Figs. 10(*c*) and 11(*c*), is less uniform.

Figures 9(*b*) and 9(*c*) relate to the rectangular pulse. The same characteristics as indicated above are evident, except for the pronounced difference that X_m/X_g is always 2.00, regardless of X_g/G.

The effect on X_m of variation in the degree of bi-linearity has been summarized in Fig. 12 where the ratio $(X_m)_{b11}/(X_m)_{11a}$ has been plotted

as a function of $G/(X_m)_{\text{lin}}$. Several generalizations may now be made based on Fig. 12:

1. When $G/(X_m)_{\text{lin}} \gtrless 1$ the system is obviously linear for all values of t_g/T_1.

2. When $G/(X_m)_{\text{lin}} = 0$ the system is linear for all values of t_g/T_1. This is exactly true if $G = 0$, and is sensibly true if X_g is large in comparison with G.

3. There is, in general, some optimum value of $G/(X_m)_{\text{lin}}$ for which the absolute maximum distortion of the bi-linear system will be a minimum. The case of the rectangular pulse is an exception.

4. As τ approaches unity the system approaches linearity, but the approach to linearity is rather slow.

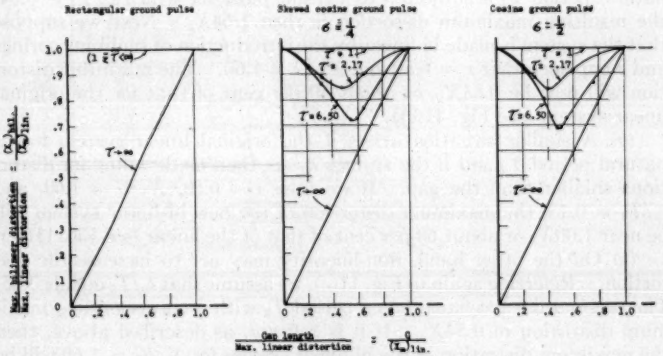
Rectangular ground pulse
Skewed cosine ground pulse
σ = 1/4
Cosine ground pulse
σ = 1/2
τ = 1
τ = 2.17
τ = 6.50
τ = ∞
Max. bi-linear distortion / Max. linear distortion = (X_m)bi-lin. / (X_m)lin.
Gap length / Max. linear distortion = G / (X_m)lin.

FIG. 12. Effect of bi-linearity τ and of gap length on maximum distortion; for three types of ground pulses.

(*a*) Rectangular ground pulse.
(*b*) Skewed cosine ground pulse, $\sigma = \frac{1}{4}$.
(*c*) Cosine ground pulse, $\sigma = \frac{1}{2}$.

5. Another extreme is possible if we make the snubbing springs k_2' infinitely stiff. This case is represented by the straight line passing through the origin and through the point (1.0, 1.0). This limiting condition indicates in Fig. 12(*b*) and (*c*) that curves for any value of τ must stay on the left side of the limiting straight lines.

The following general remarks can now be made about the bi-linear systems:

1. X/X_g may occur either during the forced vibration or the ensuing free vibration, depending on t_g/T_1, the form of the pulse, and the degree of bi-linearity.

2. $X/X_g = 1.00$ at $t_g/T_1 \to 0$, for any form of pulse and for any system.

3. X/X_g never falls below 1.00, for the rectangular pulse.

4. X_m/X_g for the linear systems of period T_1 and T_2 occur at two specific values of t_g/T_1, which depend on the form of the ground pulse. X_m/X_g for a member of the bi-linear family occurs at some value of t_g/T_1 lying between the two specific values for the limiting linear systems. Moreover, when the pulse is rectangular, X_m/X_g for a bi-linear system occurs at many values of t_g/T_1 in a manner similar to that for the linear system.

Examples

(*a*) More startling effects of non-linearity appear if we take some particular values of t_g/T_1. For example, assume that a linear system of natural period T_1 is subjected to a cosine pulse for which $t_g/T_1 = 0.84$; the resulting maximum distortion is then $1.64X_g$. Next we suppose that the system is made bi-linear by the introduction of snubbing springs and a gap such that $\tau = 6.50$ and $X_g/G = 1.60$. The maximum distortion will now be $0.83X_g$, or about 50 per cent of that for the original linear system (see Fig. 11(*c*)).

(*b*) A similar situation arises if the original linear system has a natural period T_2 and if the springs k_1 are then made softer for distortions smaller than the gap. If we take $\tau = 6.50$, $X_g/G = 1.60$, and $t_g/T_1 = 0.13$, the maximum distortion of the new bi-linear system will be near $1.00X_g$ or about 60 per cent of that of the linear (see Fig. 11(*c*)).

(*c*) On the other hand, non-linearity may act to increase the distortion. Referring again to Fig. 11(*c*), we assume that t_g/T_1 equals 0.25. The original linear system has a period T_2 with a corresponding maximum distortion of $0.54X_g$. If it is *softened*, as described above, then the maximum distortion of the bi-linear system for $X_g/G = 1.60$ will be $1.32X_g$ or about two and one-half times that of the linear case. Similarly, if the original linear system has a period T_1 and if *snubbing* springs are added, the maximum distortion of the bi-linear system for $T_g/G = 1.60$ will be nearly 70 per cent greater than that for the linear. The possibility of increased distortion due to non-linearity and particular values of t_g/T_1 is also apparent in the case of the rectangular pulse (see Fig. 9(*c*)).

Restoring Force and Acceleration

The foregoing discussion has been related to relative displacement or distortion. Criteria which often may be of equal or greater importance in judging the effect of a pulse are restoring force and acceleration.

The variation of maximum restoring force with duration of ground motion has been shown for one bi-linear family and for one type of ground pulse in Fig. 13. The absolute maximum restoring force in the limiting linear system of natural period T_2 is $(T_1/T_2)^2$, or k_2/k_1, times

the absolute maximum for the limiting system of period T_1. The absolute maxima for the intervening bi-linear systems lie between those for the linear systems. The bi-linearity results in *either* increased or decreased restoring force and acceleration, depending on particular values of t_0/T_1.

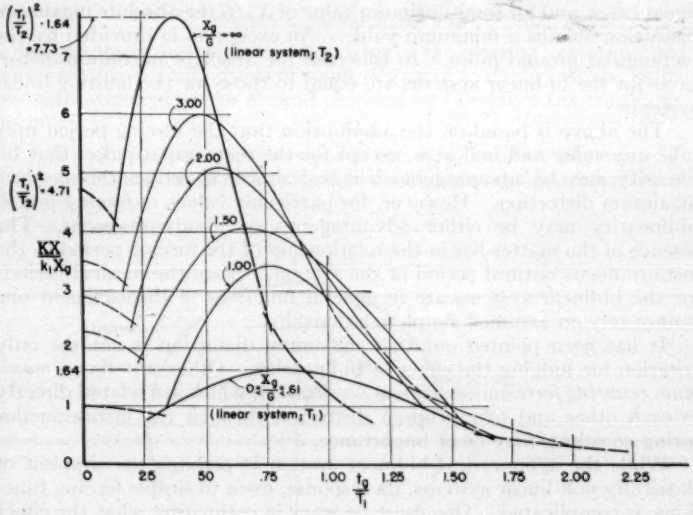

FIG. 13. Restoring force in bi-linear systems, $T_1/T_2 = \tau = 2.17$; cosine ground pulse.

SUMMARY AND CONCLUSIONS

The quantitative results of the investigation have been presented mainly in the form of envelopes of maximum distortion plotted as a function of forcing period, where the variables have been expressed in dimensionless form. Each envelope includes a number of discontinuities in slope which arise from the transient nature of the phenomenon. More specifically, for some values of forcing period the maximum distortion is the first of the maxima of the distortion-time function; for other forcing periods it is the second, and so on; this change in the order of occurrence of the maximum results in the discontinuities. The manner of determining the envelopes has been shown in detail for the linear system.

Corresponding to each family of linear and bi-linear systems there is, for a particular form of forcing function, a family of envelopes of maximum distortion. As X_0/G is increased beyond a limiting finite value,

the envelopes for the bi-linear members of the family branch off from the envelope for the limiting linear system of period T_1 and gradually approach the envelope for the limiting linear system of period T_2. The absolute maximum values of the two limiting envelopes are, of course, equal. The absolute maximum distortion of a bi-linear system, however, is in general less than the absolute maximum for the limiting linear cases, and for some optimum value of X_0/G the absolute maximum distortion attains a minimum value. An exception is provided by the rectangular ground pulse. In this case the absolute maximum distortions for the bi-linear systems are equal to those for the limiting linear systems.

The above is based on the assumption that the forcing period may take *any* value and indicates, except for the rectangular pulse, that bi-linearity may be advantageous if it is desirable to reduce the absolute maximum distortion. However, for particular values of forcing period bi-linearity may be either advantageous or disadvantageous. The essence of the matter lies in the relationship of the forcing period to the instantaneous natural period of the system. Since the natural periods for the bi-linear systems are in general functions of displacement one cannot rely on assumed simple relationships.

It has been pointed out that maximum distortion is not the only criterion for judging the effect of bi-linearity. Other criteria as *maximum restoring force* and *maximum acceleration*, which are related directly to each other and to maximum distortion through the instantaneous spring constant, may be of importance.

While the symmetrical bi-linear system is perhaps the simplest of elastically non-linear systems, its response, even to simple forcing functions, is complicated. One must be wary in estimating what the effect of bi-linearity will be. If many problems are to be solved an experimental apparatus is advantageous and can be constructed without great effort. The particular form of apparatus used in this investigation is not necessarily recommended, although it has proved satisfactory. If only a few specific problems are to be solved the graphical method is superior to the experimental.

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APPENDIX A, GRAPHICAL METHOD

The graphical method is an adaptation of the rotating vector method of representing the vibrations of single degree of freedom systems (5, 6). The adaptation relates to ground motions and involves the consideration of:

(1) The trajectory of the system in the $\dot{x}_s/p$, x_s phase plane.

(2) An approximation of the actual ground motion by a sequence of rectangular steps.

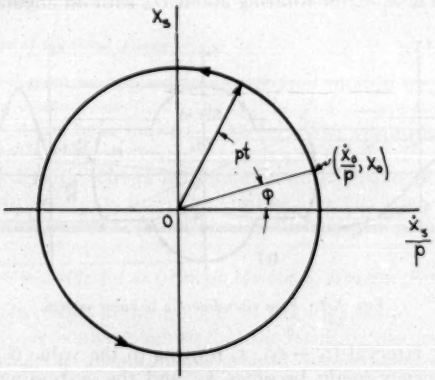

FIG. A-1. Free vibration of a linear system.

(3) The changes in the position of static and/or temporary points of equilibrium about which the system will oscillate as a result of the ground motion and/or the changes in the incremental spring constant of the non-linear systems.

(4) The sudden changes in the coordinate $\dot{x}_s/p$ as the incremental spring constant (and hence the effective natural frequency p) vary.

The presentation of the method for the bi-linear system will be facilitated by considering some simpler cases first.

Free Vibration of a Linear System

Let the initial displacement and velocity be x_0 and $\dot{x}_0$. The vibration of the system may be represented by a circle passing through the

point $\dot{x}_0/p$, x_0 and with center at the origin. The circle is generated by a vector of length equal to the amplitude of the vibration, rotating with constant velocity p. (Fig. A-1.)

Free Vibration of a Bi-Linear System

The spring characteristic of Fig. A-2(a) is similar to that of Fig. 2 except that it has been rotated through a positive angle of 90°. Let $x_0 < G$ and $\dot{x}_0 > 0$ (Fig. A-2(b)). During the first stage the system has a natural frequency p_1 and moves about the static position of equilibrium 0. After a time t_1, x_s becomes equal to G and the system starts to oscillate with an effective natural frequency p_2 about the temporary position of equilibrium O_R. Due to the sudden change in effective natural frequency the abscissa at $x_s = G$ must change suddenly from $\dot{x}_1/p_1$ to $\dot{x}_1/p_2$ or an amount $1 - 1'$. The ensuing vibration is described by a new vector rotating about O_R with an angular velocity p_2.

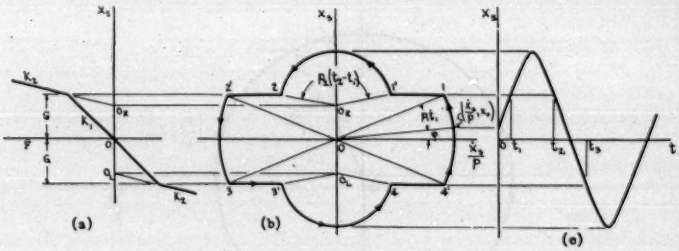

FIG. A-2. Free vibration of a bi-linear system.

After a time interval $(t_2 - t_1)$, x_s returns to the value G, the effective natural frequency again becomes p_1, and the system again oscillates about the static equilibrium position O. The process continues, the rotating vectors describing a symmetrical closed diagram as a consequence of the symmetrical spring characteristic and zero damping of the system. The representation is exact. Displacement-time and velocity-time diagrams may be constructed from the $\dot{x}_s/p$, x_s diagram as illustrated in Fig. A-2(c).

Linear System Subjected to a Rectangular Ground Pulse

Assume that the system is initially at rest and unstrained. The motion of the system until the time t_1 is represented by the circular arc $0, 0' - 1, 1'$ described by a vector of length X_g rotating (through an angle pt_1) about the displaced position of static equilibrium 0_1 (Fig. A-3(b)). At the time t_1 the position of equilibrium is returned to its

original location, and the ensuing free vibrations are described by the circle with center at 0. The ground motion and the displacement-time diagrams appear in Fig. A-3(*c*). The method is exact.

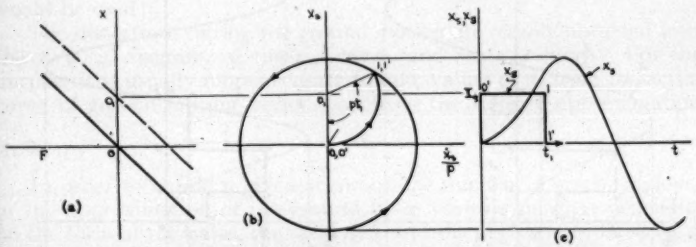

FIG. A-3. Linear system subjected to a rectangular ground pulse.

Ground Motion of General Form

The method may be applied to any ground motion by the use of a stepped approximation, as illustrated in Fig. A-4, where for clarity the number of steps has been limited to three. The vibration during the ground motion is represented by the arcs $0, 0' - 1, 1'$, $1, 1' - 2, 2'$ and $2, 2' - 3, 3'$ with centers at the temporarily displaced static positions of equilibrium 0_1, 0_2, 0_3. The ensuing free vibration is described by the circle with center at 0 and passing through the point $3, 3'$.

Bi-Linear System Subjected to Ground Motion of General Form

The first step results in the arc $0, 0' - 1$ of Fig. A-5(*b*). The ground displacement x_{g1} is small enough so that the resulting distortions are less than G. During the instantaneous ground displacement occurring at the start of the second step ($t = t_1$) the distortion is forced to a value numerically greater than G, the effective natural frequency becomes p_2 and the abscissa of the $\dot{x}_s/p$, x_s diagram at $t = t_1$ decreases from $\dot{x}_1/p_1$ to $\dot{x}_1/p_2$ or by an amount $1 - 1'$. The system starts to oscillate about the effective position of equilibrium 0_{2L} and continues so until the distortion has decreased to a value equal to G (at $t_1 < t < t_2$). At this instant the effective natural frequency changes from p_2 to p_1, the abscissa increases correspondingly, and the system starts to oscillate about the displaced position of static equilibrium 0_2; it continues so until $t = t_2$.

The vibration from time t_2 until time t_3 may be similarly explained. At $t = t_3$ the ground returns to its original position at $x_g = 0$ and remains there, and the system describes free vibrations with starting conditions determined by point $3, 3'$.

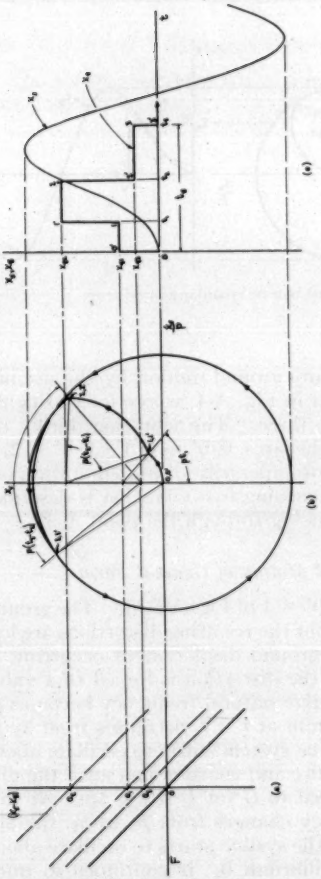

FIG. A-4. Linear system; stepwise ground motion.

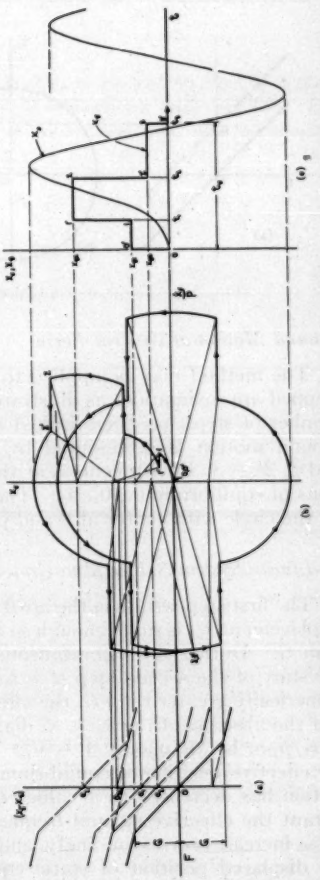

FIG. A-5. Bi-linear system; stepwise ground motion.

While the above example illustrates several important points regarding change in x from a value smaller than G to one larger than G, and *vice versa*, it is somewhat distorted by the fact that only three steps have been taken to approximate the ground pulse. Ordinarily more steps would be used.

The distortions during the ground motion are readily obtained from the $\dot{x}_s/p$, x_s diagram, or the x_s,t curve, and the x_g,t curve. For this purpose it is usually more accurate to take values of x_g from the actual curve of ground motion rather than from the stepped approximation.

Accuracy

In order to obtain a given accuracy the number of steps employed in the approximation of the ground pulse varies somewhat depending on the form of the pulse, the ratio t_g/T and the degree of bi-linearity of the system. Some idea of the accuracy obtained with a linear system is indicated below:

Cosine pulse with t_g/T corresponding to X_m:
16 equal steps 99 per cent, 8 equal steps 97 per cent, and 6 equal steps 97 per cent.

"Skewed" cosine pulse $\sigma = \frac{1}{4}$ and with t_g/T corresponding to X_m:
24 equal steps 99 per cent, 12 unequal steps 98 per cent, 8 unequal steps 96 per cent, and 6 unequal steps 94 per cent.

General Application

The method is a powerful one. In application to linear or to stepped non-linear systems its theoretical accuracy is limited only by the number of steps one is willing to use in approximating the forcing function. In application to a system with continuous non-linearity the continuous restoring force-distortion function must also be replaced by some stepped approximation (6).

The restoring force-distortion function may be asymmetrical instead of symmetrical and may include portions with negative slope (8). Damping may be treated by continuously modifying the length of the rotating vector according to the law of damping (6). There is no restriction on the starting conditions.

APPENDIX B, EXPERIMENTAL APPARATUS

The experimental apparatus consisted of a translatory mechanical system subjected to transient ground motions by means of a cam-driven mechanical linkage as shown in Fig. B-1. The cam C, shaped to generate the ground pulse, is driven at constant velocity, variable over a wide range. The pulse is imparted to the ground structure GS through the cam follower F and linkage A, L. A restoring spring RS

maintains continuous contact between the follower and the cam. The natural frequency of the linkage, ground structure and restoring spring system is high relative to the operating frequencies. The wire W is practically inextensible. The mass of the upper spring system, S_2, W,

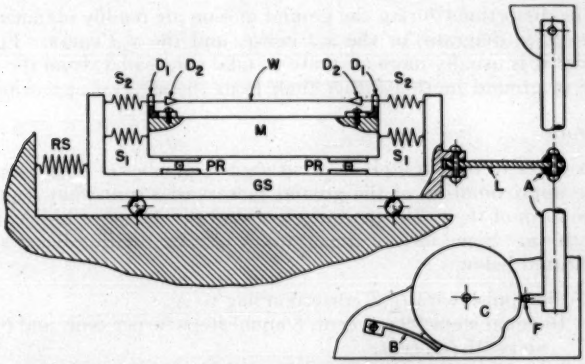

FIG. B-1. Schematic diagram of experimental apparatus.

C	cam	A, L	linkage
F	cam follower	$G S$	ground structure
B	brake	$R S$	restoring spring
M	mass		
S_1	springs of combined constant k_1		
S_2	springs of combined constant k_2		
W	wire, connecting springs S_2		
D_1	adjustable stops mounted on M		
D_2	stops attached to W		
PR	glass plate—steel roller supports		

D_2, is very small in comparison with the mass M; therefore, kinetic energy transfers to the upper system are negligible. Friction was minimized by the use of the glass plate-steel roller supports PR under the mass. The displacements x and x_g were recorded continuously by a direct-inking, mechanical device.

A MODERN APPROACH TO "DIMENSIONS"

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

A characteristic of nineteenth-century physics was the belief that the entire subject could be based on the laws of classical mechanics. These principles had been found applicable over a tremendous range, from astronomical values of 10^{12} m to molecular dimensions of 10^{-10} m. The apparently foreign subject of heat had been made a branch of mechanics by means of kinetic theory. It was felt that further work would undoubtedly make electromagnetism another branch of mechanics.

Under these circumstances, it was natural that the concepts of length, mass, and time, which had been chosen as fundamental in mechanics, should be applied to all branches of physics. But the twentieth century brought quantum mechanics and relativity, which practically destroyed the mechanistic idea of the universe. As a result, the feeling that everything had to be expressed in terms of the primary "dimensions" l, m, t was abandoned by most physicists. Bridgman (1) [3] emphasized the fact that there is nothing sacrosanct about l, m, t and that "dimensional" analysis is merely a man-made tool that may be manipulated at will. This principle of free choice of the primary concepts has been widely accepted, though one still finds references to "true" or "absolute dimensions" and statements that the "real dimensions" of ϵ and μ are not yet known.

After having accepted the principle of relativity of "dimensions," one is faced by the question of what choice of primary concepts will be of maximum utility. This is the subject of the present paper.

1. FUNDAMENTALS

As pointed out previously (2), the geometric approach to "dimension" theory has advantages. A complete physical quantity [4] possesses both magnitude and "quality" and is thus similar to a vector. The *magnitude* of the physical quantity is represented by the length of the vector, while the *kind* of physical quantity is represented by the direction of the vector.

[1] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[2] Brown University, Providence, R. I.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

[4] A scalar quantity or one component of a vector in euclidean 3-space. Note that the "vector" referred to in the above sentence of the text is an *idon* vector.

We write one of these "dimensional" vectors as a symbol inclosed in curly brackets, as $\{A\}$. The magnitude is denoted by A and the direction is specified by the *designation symbol* $[a^1, a^2, \cdots a^k]$. Thus,

$$\{A\} = A[a^1, a^2, \cdots a^k] = Aa^i, \qquad (i = 1, 2, \cdots k).$$

Geometrically, $\{A\}$ is a vector in a k-dimensional affine space, A is a scalar, and a^i is a *base vector* which fixes the direction of $\{A\}$. To distinguish $\{A\}$ from vectors in the euclidean space in which the physical phenomena occur, $\{A\}$ is called an *idon*. The fictitious k-space of the idon vectors is called *idon space*. The *coordinates* $a^1, a^2, \cdots a^k$ of the base vector are equal to the exponents of the usual "dimensional" formulas.

Allowable operations on idons are as follows (2):

(*1*) *Addition*. The addition of two idons is possible only if they are collinear. This corresponds to the well-known principle of "dimensional" homogeneity. In this case, the sum of

$$\{A\} = Aa^i \qquad \text{and} \qquad \{B\} = Bb^i$$

is defined as

$$\{A\} + \{B\} = (A + B)a^i = (A + B)b^i. \tag{1}$$

(*2*) *Multiplication by a scalar*. Let k be an *idonic scalar* ("dimensionless" quantity). Then the product of it and an idon is defined as

$$k\{A\} = kAa^i = Aka^i. \tag{2}$$

(*3*) *Multiplication*. Multiplication of two idons is defined by the equation,

$$\{A\}\{B\} = AB[a^i + b^i]. \tag{3}$$

Note that this product is similar to the familiar product of two complex quantities in polar form, where the lengths are multiplied and the angles added. Here the corresponding coordinates of the base vectors are added in taking the product.

Repeated multiplication leads to the equation,

$$\{A\}^\gamma = (A)^\gamma[\gamma a^i] \tag{4}$$

where γ may be extended to non-integer values if desired.

(*4*) *Division*. Division of idons is defined as

$$\frac{\{A\}}{\{B\}} = \frac{A}{B}[a^i - b^i]. \tag{5}$$

An example is shown in Fig. 1. The primary concepts are chosen as length, mass, and time so $k = 3$ and the coordinate axes are l, m, t. Any length is written

$$\{l\} = l[1, 0, 0],$$

any mass is

$$\{m\} = m[0, 1, 0],$$

and any time is

$$\{t\} = t[0, 0, 1].$$

Any equation of physics can be rewritten in idon form by merely replacing the usual quantities by idons. For instance, velocity is defined as

$$v = dl/dt.$$

In idon form,

$$\{v\} = \frac{d\{l\}}{d\{t\}}$$

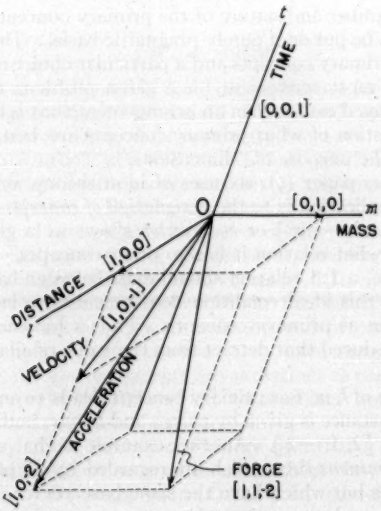

FIG. 1. Division of l and t idons result in idons for v and a.

or, since the base vectors are invariant with respect to the differentiation,

$$v[a^1, a^2, a^3] = \frac{dl}{dt}\frac{[1, 0, 0]}{[0, 0, 1]} = \frac{dl}{dt}[1, 0, -1].$$

For equality, both magnitude and direction must be the same, giving the two equations,

$$v = dl/dt \quad \text{and} \quad [a^1, a^2, a^3] = [1, 0, -1].$$

Similarly, acceleration has the base vector $[1, 0, -2]$. Any accelera-

tion is therefore represented by a vector in the direction of the acceleration base vector (Fig. 1).

If we define *force* as

$$F = ma,$$

then

$$\{F\} = \{m\}\{a\} = ma([0, 1, 0] + [1, 0, -2])$$
$$= ma[1, 1, -2].$$

Thus the base vector associated with the concept of force is $[1, 1, -2]$, if l, m, t are chosen as primary concepts.

2. AMBIGUITY

Since the number and nature of the primary concepts are arbitrary, their choice can be put on a purely pragmatic basis. That is, a particular number of primary concepts and a particular choice of concepts may turn out to be most convenient for a given problem, and this choice should be employed rather than an arrangement that is less useful. To answer the question of what primary concepts are best, therefore, one must consider the purpose of "dimensions."

In a previous paper (3), six uses of idon theory were listed. The fundamental application is to the *designation of concepts*. The *designation symbol* $[a^1, a^2, \cdots a^k]$ or *base vector* shows at a glance what the concept is and what relation it has to other concepts. If the method is to be effective, a 1:1 relation should exist between base vectors and concepts. But this ideal condition does not obtain, in general, when l, m, t are chosen as primary concepts. Thus a looseness and an ambiguity are introduced that detract from the value of all six applications of idon theory.

That the use of l, m, t as primary concepts leads to ambiguity is well known. One instance is given by *energy* and *torque*, both of which have the base vector $[2, 1, -2]$. Another example is that of *luminous intensity* and *luminous flux*, which are regarded by all photometrists as distinct concepts but which have the same base vector.

In electricity and magnetism, the same concept has different designation symbols depending on whether the "electrostatic" or the "electromagnetic" cgs. system is employed. Moreover, some of the base vectors assigned to electric and magnetic concepts are already assigned to familiar mechanical concepts. In the "electrostatic" system, for instance, *capacitance* has the same base vector as *length*, *resistivity* has the same base vector as *time*, *resistance* is equivalent "dimensionally" to reciprocal velocity, and *permeability* is equivalent to reciprocal velocity squared. In the "electromagnetic" system, *inductance* and *length* have the same base vector, as do *resistance* and *velocity*.

To talk of inductance and capacitance in *centimeters*, resistivity in *seconds*, and resistance in *centimeters per second* is not wrong but it is

confusing, and only our long familiarity with such "dimensional" monstrosities makes us willing to tolerate them. Physicists in general have learned to take these discrepancies in their stride and to consider them as normal. For instance, Birge (4) says, ". . . one cannot, from the adopted dimensions of a quantity, draw any conclusions as to its physical nature; thus two quantities of quite different character, such as moment of force and energy, have, in the cgs. system, the same dimensions."

The point is that this "dimensional" ambiguity is not a fact of nature. It is merely an imperfection in our man-made scheme of assigning primary concepts. If idon theory is to be of maximum serviceability, it must be revised so that each concept has its unique base vector.

3. THE *l*, *m*, *t* SYSTEM

Table I lists several possible designation systems for the concepts of physics and engineering. The second column indicates the result of choosing the traditional *l*, *m*, and *t* as primary concepts. For mechanics, the designation symbols are well known and are fairly satisfactory. The most notable ambiguity occurs with *energy* and *torque*, as mentioned in the previous section. Despite the identity in "dimensions," physicists always regard energy and torque as distinct concepts. Another example is the confusion among *normal stress*, *shearing stress*, *pressure*, and *elastic moduli*, all having the base vector $[-1, 1, -2]$. These concepts are often considered identical, from the mistaken notion that "dimensions" are associated with units instead of concepts and that identical *units* imply identical *concepts* (3). From an operational standpoint, however, the above concepts are as distinct as energy and torque. It would seem preferable, therefore, to associate normal stress and shearing stress with different base vectors.

Turning now to the subject of *heat*, we find that *l*, *m*, *t* can still be employed, but an assumption must be made about the "dimensionality" of some constant. One suggestion (5) has been to base the analysis on the equation,

$$pv = RT$$

and to assume that R has the same designation symbol as mass. This assumption leads to the interesting result that *entropy* and *mass* are "dimensionally" identical and that *temperature* has the same base vector as velocity squared. These ambiguities are listed in Table II.

There is, of course, no necessity that this particular assumption be made. Condon (6) takes the Wien displacement relation, $\lambda T = \text{const.}$ and assumes that the constant is an idonic scalar. The results of this proposal are marked (*b*) in the second column of Table I. Ambiguities are found between *temperature* and *reciprocal length*, *thermal power* and

TABLE I.—*Designation Symbols for Physical Concepts.*

Name	[*l*, *m*, *t*]		[*l*, *m*, *t*, *T*, *Q*, *F*]	[*l*, *t*, *P*, *T*, *Q*, *F*]
Geometry				
Distance	[1, 0, 0]		[1, 0, 0, 0, 0, 0]	[1, 0, 0, 0, 0, 0]
Area	[2, 0, 0]		[2, 0, 0, 0, 0, 0]	[2, 0, 0, 0, 0, 0]
Volume	[2, 0, 0]		[3, 0, 0, 0, 0, 0]	[3, 0, 0, 0, 0, 0]
Angle	[0]		[0]	[0]
Solid angle	[0]		[0]	[0]
Kinematics				
Time	[0, 0, 1]		[0, 0, 1, 0, 0, 0]	[0, 1, 0, 0, 0, 0]
Velocity	[1, 0, −1]		[1, 0, −1, 0, 0, 0]	[1, −1, 0, 0, 0, 0]
Acceleration	[1, 0, −2]		[1, 0, −2, 0, 0, 0]	[1, −2, 0, 0, 0, 0]
Angular velocity	[0, 0, −1]		[0, 0, −1, 0, 0, 0]	[0, −1, 0, 0, 0, 0]
Angular acceleration	[0, 0, −2]		[0, 0, −2, 0, 0, 0]	[0, −2, 0, 0, 0, 0]
Frequency	[0, 0, −1]		[0, 0, −1, 0, 0, 0]	[0, −1, 0, 0, 0, 0]
Dynamics				
Force	[1, 1, −2]		[1, 1, −2, 0, 0, 0]	[−1, 1, 1, 0, 0, 0]
Momentum	[1, 1, −1]		[1, 1, −1, 0, 0, 0]	[−1, 2, 1, 0, 0, 0]
Impulse	[1, 1, −1]		[1, 1, −1, 0, 0, 0]	[−1, 2, 1, 0, 0, 0]
Torque	[2, 1, −2]		[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Energy	[2, 1, −2]		[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Power	[2, 1, −3]		[2, 1, −3, 0, 0, 0]	[0, 0, 1, 0, 0, 0]
Mass	[0, 1, 0]		[0, 1, 0, 0, 0, 0]	[−2, 3, 1, 0, 0, 0]
Moment of inertia	[2, 1, 0]		[2, 1, 0, 0, 0, 0]	[0, 3, 1, 0, 0, 0]
Dissipation coefficient	[0, 1, −1]		[0, 1, −1, 0, 0, 0]	[−2, 2, 1, 0, 0, 0]
Stiffness	[0, 1, −2]		[0, 1, −2, 0, 0, 0]	[−2, 1, 1, 0, 0, 0]
Efficiency	[0]		[0]	[0]
Elasticity				
Stress, normal	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
shear	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Strain, normal	[0]		[0]	[0]
shear	[0]		[0]	[0]
Elastic moduli	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Hydrodynamics				
Pressure	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Velocity potential	[2, 0, −1]		[2, 0, −1, 0, 0, 0]	[2, −1, 0, 0, 0, 0]
Density	[−3, 1, 0]		[−3, 1, 0, 0, 0, 0]	[−5, 3, 1, 0, 0, 0]
Viscosity	[−1, 1, −1]		[−1, 1, −1, 0, 0, 0]	[−3, 2, 1, 0, 0, 0]
Surface tension	[0, 1, −2]		[0, 1, −2, 0, 0, 0]	[−2, 1, 1, 0, 0, 0]
Heat				
	[*l*, *m*, *t*]	[*l*, *m*, *t*]		
	a	*b*		
Temperature	[2, 0, −2]	[−1, 0, 0]	[0, 0, 0, 1, 0, 0]	[0, 0, 0, 1, 0, 0]
Thermal energy (heat)	[2, 1, −2]	[2, 1, −2]	[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Thermal power	[2, 1, −3]	[2, 1, −3]	[2, 1, −3, 0, 0, 0]	[0, 0, 1, 0, 0, 0]
Entropy	[0, 1, 0]	[3, 1, −2]	[2, 1, −2, −1, 0, 0]	[0, 1, 1, −1, 0, 0]

TABLE I.—*Continued.*

Name	[*l*, *m*, *t*]		[*l*, *m*, *t*, *T*, *Q*, *F*]	[*l*, *t*, *P*, *T*, *Q*, *F*]
Power per unit volume	[−1, 1, −3]	[−1, 1, −3]	[−1, 1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Thermal conductivity	[−1, 1, −1]	[+2, +1, −3]	[1, 1, −3, −1, 0, 0]	[−1, 0, 1, −1, 0, 0]
Thermitivity (spec. ht.)	[0]	[3, 0, −2]	[2, 0, −2, −1, 0, 0]	[2, −2, 0, −1, 0, 0]
Diffusivity	[2, 0, −1]	[2, 0, −1]	[2, 0, −1, 0, 0, 0]	[2, −1, 0, 0, 0, 0]
Thalpance ("emissivity")	[−2, 1, −1]	[1, 1, −3]	[0, 1, −3, −1, 0, 0]	[−2, 0, 1, −1, 0, 0]
Coefficient of linear expansion	[−2, 0, 2]	[+1, 0, 0]	[0, 0, 0, −1, 0, 0]	[0, 0, 0, −1, 0, 0]
Thermoelectric coefficient	[$-\frac{3}{2}$, $\frac{1}{2}$, 1][a]	[$\frac{3}{2}$, $\frac{1}{2}$, −1][a]	[2, 1, −2, −1, −1, 0]	[0, 1, 1, −1, −1, 0]
Temp. coefficient of resistance	[−2, 0, 2]	[+1, 0, 0]	[0, 0, 0, −1, 0, 0]	[0, 0, 0, −1, 0, 0]
Electrostatics	[a]	[e]		
Charge	[$\frac{3}{2}$, $\frac{1}{2}$, −1]	[$\frac{1}{2}$, $\frac{1}{2}$, 0]	[0, 0, 0, 0, 1, 0]	[0, 0, 0, 0, 1, 0]
Charge per unit length	[$\frac{1}{2}$, $\frac{1}{2}$, −1]	[$-\frac{1}{2}$, $\frac{1}{2}$, 0]	[−1, 0, 0, 0, 1, 0]	[−1, 0, 0, 0, 1, 0]
Charge per unit area	[$-\frac{1}{2}$, $\frac{1}{2}$, −1]	[$-\frac{3}{2}$, $\frac{1}{2}$, 0]	[−2, 0, 0, 0, 1, 0]	[−2, 0, 0, 0, 1, 0]
Charge per unit volume	[$-\frac{3}{2}$, $\frac{1}{2}$, −1]	[$-\frac{5}{2}$, $\frac{1}{2}$, 0]	[−3, 0, 0, 0, 1, 0]	[−3, 0, 0, 0, 1, 0]
Electric field strength	[$-\frac{1}{2}$, $\frac{1}{2}$, −1]	[$\frac{1}{2}$, $\frac{1}{2}$, −2]	[1, 1, −2, 0, −1, 0]	[−1, 1, 1, 0, −1, 0]
Electric potential (and pot. difference)	[$\frac{1}{2}$, $\frac{1}{2}$, −1]	[$\frac{3}{2}$, $\frac{1}{2}$, −2]	[2, 1, −2, 0, −1, 0]	[0, 1, 1, 0, −1, 0]
Electric flux density	[$-\frac{1}{2}$, $\frac{1}{2}$, −1]	[$-\frac{3}{2}$, $\frac{1}{2}$, 0]	[−2, 0, 0, 0, 1, 0]	[−2, 0, 0, 0, 1, 0]
Electric flux	[$\frac{3}{2}$, $\frac{1}{2}$, −1]	[$\frac{1}{2}$, $\frac{1}{2}$, 0]	[0, 0, 0, 0, 1, 0]	[0, 0, 0, 0, 1, 0]
Permittivity	[0]	[−2, 0, 2]	[−3, −1, 2, 0, 2, 0]	[−1, −1, −1, 0, 2, 0]
Capacitance	[1, 0, 0]	[−1, 0, 2]	[−2, −1, 2, 0, 2, 0]	[0, −1, −1, 0, 2, 0]
Electric Current	[a]	[e]		
Current	[$\frac{3}{2}$, $\frac{1}{2}$, −2]	[$\frac{1}{2}$, $\frac{1}{2}$, −1]	[0, 0, −1, 0, 1, 0]	[0, −1, 0, 0, 1, 0]
Current density	[$-\frac{1}{2}$, $\frac{1}{2}$, −2]	[$-\frac{3}{2}$, $\frac{1}{2}$, −1]	[−2, 0, −1, 0, 1, 0]	[−2, −1, 0, 0, 1, 0]
Resistivity	[0, 0, 1]	[2, 0, −1]	[3, 1, −1, 0, −2, 0]	[1, 2, 1, 0, −2, 0]
Resistance	[−1, 0, 1]	[1, 0, −1]	[2, 1, −1, 0, −2, 0]	[0, 2, 1, 0, −2, 0]
Magnetism	[d]	[e]		
Magnetic flux density	[$-\frac{3}{2}$, $\frac{1}{2}$, 0]	[$-\frac{1}{2}$, $\frac{1}{2}$, −1]	[0, 1, −1, 0, −1, 0]	[−2, 2, 1, 0, −1, 0]
Magnetic flux	[$\frac{1}{2}$, $\frac{1}{2}$, 0]	[$\frac{1}{2}$, $\frac{1}{2}$, −1]	[2, 1, −1, 0, −1, 0]	[0, 2, 1, 0, −1, 0]
Magnetic field strength	[$\frac{1}{2}$, $\frac{1}{2}$, −2]	[$-\frac{1}{2}$, $\frac{1}{2}$, −1]	[−1, 0, −1, 0, 1, 0]	[−1, −1, 0, 0, 1, 0]

TABLE I.—*Continued.*

Name	[*l*, *m*, *t*]		[*l*, *m*, *t*, *T*, *Q*, *F*]	[*l*, *t*, *P*, *T*, *Q*, *F*]
Permeability	[−2, 0, 2]	[0]	[1, 1, 0, 0, −2, 0]	[−1, 3, 1, 0, −2, 0]
Magnetomotive force	[½, ½, −2]	[½, ½, −1]	[0, 0, −1, 0, 1, 0]	[0, −1, 0, 0, 1, 0]
Vector potential	[−½, ½, 0]	[½, ½, −1]	[1, 1, −1, 0, −1, 0]	[−1, 2, 1, 0, −1, 0]
Inductance	[−1, 0, 2]	[1, 0, 0]	[2, 1, 0, 0, −2, 0]	[0, 3, 1, 0, −2, 0]
Poynting vector	[0, 1, −3]	[0, 1, −3]	[0, 1, −3, 0, 0, 0]	[−2, 0, 1, 0, 0, 0]
Radiometry				
Radiant pharos (radiant power)	[2, 1, −3]		[2, 1, −3, 0, 0, 0]	[0, 0, 1, 0, 0, 0]
Radiant pharosage	[0, 1, −3]		[0, 1, −3, 0, 0, 0]	[−2, 0, 1, 0, 0, 0]
Radiant pharosum	[−1, 1, −3]		[−1, 1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Radiant phos (energy)	[2, 1, −2]		[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Radiant phosage	[0, 1, −2]		[0, 1, −2, 0, 0, 0]	[−2, 1, 1, 0, 0, 0]
Radiant phosum	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Radiant helios	[0, 1, −3]		[0, 1, −3, 0, 0, 0]	[−2, 0, 1, 0, 0, 0]
Radiant heliosent	[−1, 1, −3]		[−1, +1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Phengosage	[−1, 1, −3]		[−1, 1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Photometry	*f*			
Luminous pharos (luminous flux)	[2, 1, −3]		[0, 0, 0, 0, 0, 1]	[0, 0, 0, 0, 0, 1]
Luminous pharosage	[0, 1, −3]		[−2, 0, 0, 0, 0, 1]	[−2, 0, 0, 0, 0, 1]
Luminous pharosum	[−1, 1, −3]		[−3, 0, 0, 0, 0, 1]	[−3, 0, 0, 0, 0, 1]
Luminous phos ("light")	[2, 1, −2]		[0, 0, 1, 0, 0, 1]	[0, 1, 0, 0, 0, 1]
Luminous phosage	[0, 1, −2]		[−2, 0, 1, 0, 0, 1]	[−2, 1, 0, 0, 0, 1]
Luminous phosum	[−1, 1, −2]		[−3, 0, 1, 0, 0, 1]	[−3, 1, 0, 0, 0, 1]
Luminous helios	[0, 1, −3]		[−2, 0, 0, 0, 0, 1]	[−2, 0, 0, 0, 0, 1]
Luminous heliosent	[−1, 1, −3]		[−3, 0, 0, 0, 0, 1]	[−3, 0, 0, 0, 0, 1]
Lamprosity ("visibility")	[0]		[−2, −1, 3, 0, 0, 1]	[0, 0, −1, 0, 0, 1]

[a] On the assumption that $[R]$ has base vector $[0, 1, 0]$ in $p\mathfrak{V} = RT$.
[b] On the assumption that c is an idonic scalar in $[\lambda][T] = c$.
[c] "Electrostatic" system.
[d] "Electrostatic" system. Assumption that ϵ_0 is an idonic scalar.
[e] "Electromagnetic" system. Assumption that μ_0 is an idonic scalar.
[f] Assuming that lamprosity ("visibility") is an idonic scalar. The word endings have the following meanings: −ENT = per unit length, −AGE = per unit area, −UM = per unit volume.

thermal conductivity. Still other assumptions are possible, leading to other sets of designation symbols.

In electromagnetism, conditions are even worse than in heat. The classical arrangement employs "electrostatic" and "electromagnetic" systems, and the same concept has different designation symbols in each system. The "electrostatic" system is based on the arbitrary assumption that the permittivity of free space is an idonic scalar. The "electromagnetic" system, on the other hand, employs the hypothesis that the permeability of free space is an idonic scalar.

Evidently, both these systems are arbitrary. In adding electromagnetic theory to mechanics, one must either make an arbitrary assumption about the "dimensionality" of a constant or select a fourth primary concept. The old idea that mechanics dominates all science shows itself in the choice of the former alternative, while the more modern approach consists in the selection of a new primary concept such as an *electric charge*.

As shown in Table II, the classical "electrostatic" system does not distinguish idonically between *capacitance* and *length*, or between *resistivity* and *time*, though no one believes these concepts to be anything but distinct. In the "electromagnetic" system, one finds such peculiarities as *inductance* and *length* with the same base vector. Another strange property is that in the "electrostatic" system one can distinguish idonically between B and H but not between D and E, while the converse is true in the "electromagnetic" system.

In the radiometric and photometric branches of physics, one finds ambiguities. Ordinarily the *lamprosity* (relative "visibility" of radiant energy) is assumed to be an idonic scalar, which gives to the photometric concepts the same designation symbols as those assigned to the corresponding radiometric concepts. Table II indicates also that *heliosent* and *phengosage* are idonically the same, though the concepts are completely different. Additional confusion is caused by the fact that *pharosage*, *helios*, and the *Poynting vector* are "dimensionally" indistinguishable, all having the base vector $[0, 1, -3]$. Table II is based on the modern treatment of photometrics which abandons the antiquated concept of *intensity* (candlepower). If that concept is retained, there is an additional confusion between *luminous pharos* (expressed in lumens) and *luminous intensity* (expressed in candles), and there is a similar confusion between the analogous radiometric concepts.

4. ADDITIONAL PRIMARY CONCEPTS

In the preceding section, l, m, t were considered as primary concepts. A more satisfactory system is obtained by introducing an additional primary concept for each branch of physics beyond mechanics. To eliminate the ambiguities in the subject of heat, we introduce temperature T as a primary concept. Charge Q as a primary concept helps to

TABLE II.—*Ambiguities, lmt System.*

Concepts*	Designation Symbol
Mechanics	
Energy, torque	[2, 1, −2]
Momentum, impulse	[1, 1, −1]
Pressure, normal stress, shear stress, elastic moduli	[−1, 1, −2]
Stiffness, surface tension	[0, 1, −2]
Frequency, angular velocity	[0, 0, −1]
Heat (*a*)	
Entropy, mass	[0, 1, 0]
Temperature, velocity squared	[2, 0, −2]
Thermal conductivity, viscosity	[−1, 1, −1]
Diffusivity, velocity potential	[2, 0, −1]
Thermitivity, permittivity (*d*), lamprosity, angle, solid angle, efficiency, normal strain, shear strain, permeability (*e*)	[0, 0, 0]
Heat (*b*)	
Temperature, reciprocal length	[−1, 0,0]
Diffusivity, velocity potential	[2, 0, −1]
Coefficient of linear expansion, coefficient of resistance, length	[1, 0, 0]
Thermal conductivity, thermal power	[2, 1, −3]
Electromagnetism (*d*)	
Charge, electric flux	[½, ½, −1]
Charge per unit length, electric potential	[½, ½, −1]
Charge per unit area, electric field strength, electric flux density	[−½, ½, −1]
Capacitance, length	[1, 0, 0]
Resistivity, time	[0, 0, 1]
Current, mmf.	[½, ½, −1]
Inductance, reciprocal acceleration	[−1, 0, 2]
Permeability, reciprocal temperature (*a*)	[−2, 0, 2]
Resistance, reciprocal velocity	[−1, 0, 1]
Magnetism (*e*)	
Inductance, length	[1, 0, 0]
Resistivity, diffusivity, velocity potential	[2, 0, −1]
Resistance, velocity	[1, 0, −1]
Charge, electric flux	[½, ½, 0]
Charge per unit area, electric flux density	[−½, ½, 0]
Magnetic flux density, mag. field strength	[−½, ½, −1]
Current, mmf., vector potential	[½, ½, −1]
Capacitance, reciprocal acceleration	[−1, 0, 2]
Magnetic flux, thermoelectric coefficient	[½, ½, −1]
Radiometry and Photometry	
Radiant pharos, luminous pharos, thermal conductivity (*b*)	[2, 1, −3]
Radiant pharosage, radiant helios, luminous pharosage, luminous helios, Poynting vector	[0, 1, −3]
Radiant phos, luminous phos, torque	[2, 1, −2]
Radiant phosum, luminous phosum, pressure	[−1, 1, −2]
Radiant phosage, luminous phosage, surface tension	[0, 1, −2]
Radiant heliosent, radiant pharosum, luminous heliosent, luminous pharosum, phengosage	[−1, 1, −3]

* Italic letters in parentheses refer to the footnotes to Table I.

eliminate ambiguity in electricity and magnetism, while the treatment of luminous pharos F as a basic quantity makes possible idonic discrimination between radiometric and photometric concepts.

Table III indicates the discrepancies that remain in the *lmtTQF*

TABLE III.—*Ambiguities, lmtTQF and lltPTQF System.*

Concepts	Designation Symbols	
	lmtTQF	*lltPTQF*
Mechanics		
Energy, torque	[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Momentum, impulse	[1, 1, −1, 0, 0, 0]	[−1, 2, 1, 0, 0, 0]
Normal stress, shear stress, Young's modulus, shear modulus, bulk modulus, pressure	[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Angle, solid angle, efficiency, shear strain, normal strain	[0]	[0]
Stiffness, surface tension	[0, 1, −2, 0, 0, 0]	[−2, 1, 1, 0, 0, 0]
Frequency, angular velocity	[0, 0, −1, 0, 0, 0]	[0, −1, 0, 0, 0, 0]
Heat		
Diffusivity, velocity potential	[2, 0, −1, 0, 0, 0]	[2, −1, 0, 0, 0, 0]
Coefficient of linear expansion, temperature coefficient of resistance	[0, 0, 0, −1, 0, 0]	[0, 0, 0, −1, 0, 0]
Electromagnetism		
Charge, electric flux	[0, 0, 0, 0, 1, 0]	[0, 0, 0, 0, 1, 0]
Charge per unit area, electric flux density	[−2, 0, 0, 0, 1, 0]	[−2, 0, 0, 0, 1, 0]
Current, mmf.	[0, 0, −1, 0, 1, 0]	[0, −1, 0, 0, 1, 0]
Radiometry and Photometry		
Radiant pharosage, radiant helios, Poynting vector	[0, 1, −3, 0, 0, 0]	[−2, 0, 1, 0, 0, 0]
Radiant heliosent, radiant pharosum, phengosage	[−1, 1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Luminous pharosage, luminous helios	[−2, 0, 0, 0, 0, 1]	[−2, 0, 0, 0, 0, 1]
Luminous heliosent, luminous pharosum	[−3, 0, 0, 0, 0, 1]	[−3, 0, 0, 0, 0, 1]

system. The ambiguities in heat have been almost eliminated by the introduction of T as a basic quantity. But discrepancies still remain in mechanics, electromagnetism, radiometry, and photometry.

There is also the possibility of replacing l, m, or t by other concepts considered as primary. An example is included in Table I, where power P is employed as a basic quantity instead of mass. In mechanics, this choice has no particular advantage, some of the idonic equations being more complicated and others being simpler than in the *lmtTQF* system. In heat, there is a slight advantage in the *lltPTQF* system; but the principal value occurs in radiometry, where the use of P emphasizes the parallelism between the radiometric and photometric concepts. As shown in Table III, the discrepancies are exactly the same whether P or m is used as a primary concept.

5. DIRECTED LENGTHS

Tables I to III show that the ambiguities in the *lmt* system can be greatly reduced by adding a new primary concept for each branch of physics beyond mechanics. But ambiguities still remain, even in mechanics. They occur when the concept requires the measurement of length in specific directions. In the concept of energy ("work"), for instance, length is measured in the direction of the force; while with torque, length is measured in a direction perpendicular to the force. This is the characteristic that distinguishes the two concepts, and it is a characteristic that is ignored by the systems of Table I. Consider now the possibility of distinguishing between distances in different directions.

This subject was studied by Williams (7) in 1892. In place of l, he used three fundamental lengths X, Y, Z. A set of orthogonal axes is set up for each problem and is oriented arbitrarily. The same unit of length is usually employed for X, Y, and Z, but the three concepts are considered as distinct. In this way, energy has the designation symbol X^2mt^{-2} while torque is $XYmt^{-2}$, normal stress is $XY^{-1}Z^{-1}mt^{-2}$ while shear stress is $Z^{-1}mt^{-2}$, angle is $X^{-1}Y$ and solid angle is $X^{-2}YZ$. Thus ambiguities are eliminated (8).

In some cases the additional discriminatory ability offered by the Williams system is an embarrassment rather than a help. In specifying length, for instance, one often has no need of designating a particular direction. Williams is forced to write "X or Y or Z" as designation symbol for length. Similarly, area is specified as YZ or XZ or XY, energy is X^2mt^{-2}, Y^2mt^{-2}, or Z^2mt^{-2}, with similar lack of simplicity for other concepts.

It seems that three fundamental lengths are unnecessary in idonic specification. Accordingly, we have retained only two (9), which we have called l_r and l_t. Furthermore, to obtain maximum simplicity, we have formulated a set of rules that specify whether a given length is to be called l_r or l_t. These rules are as follows:

(1) If a single direction is considered, call it l_r.

(2) If there is an area, then distances measured in the plane that is tangent to the area are called l_t, and distance perpendicular to l_t is called l_r.

(3) If there is an axis or a center, radial distance is called l_r and distance perpendicular to l_r is called l_t.

Of course these rules are arbitrary and may be replaced by any rules that the user prefers. Unless a definite convention of some kind is employed, however, it will be impossible to obtain a simple idonic designation of physical concepts. Table IV lists designation symbols on the basis of the foregoing rules. In most cases the idonic representation results from a routine application of the above ideas, though sometimes a little care must be exercised.

TABLE IV.—*Designation Symbols for Physical Concepts.*
(Scalar formulation, l_r and l_t taken as basic lengths.)

Name	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, m, t, T, \lambda, Q, F]$
Geometry			
Distance	s	l_r	[1, 0, 0, 0, 0, 0, 0, 0]
Area	$A = s_t^2$	l_t^2	[0, 2, 0, 0, 0, 0, 0, 0]
Volume	$\mathcal{V} = As_r$	$l_r l_t^2$	[1, 2, 0, 0, 0, 0, 0, 0]
Angle	$\theta = s_t/s_r$	$l_t l_r^{-1}$	[−1, 1, 0, 0, 0, 0, 0, 0]
Solid angle	$\Omega = A/r^2$	$l_t^2 l_r^{-2}$	[−2, 2, 0, 0, 0, 0, 0, 0]
Kinematics			
Time	t	t	[0, 0, 0, 1, 0, 0, 0, 0]
Velocity	$v = ds/dt$	$l_r t^{-1}$	[1, 0, 0, −1, 0, 0, 0, 0]
Acceleration	$a = dv/dt$	$l_r t^{-2}$	[1, 0, 0, −2, 0, 0, 0, 0]
Angular velocity	$\omega = d\theta/dt$	$l_t l_r^{-1} t^{-1}$	[−1, 1, 0, −1, 0, 0, 0, 0]
Angular acceleration	$\alpha = d\omega/dt$	$l_t l_r^{-1} t^{-2}$	[−1, 1, 0, −2, 0, 0, 0, 0]
Frequency	$= 1/t$	t^{-1}	[0, 0, 0, −1, 0, 0, 0, 0]
Dynamics			
Force	$F = ma$	$l_r m t^{-2}$*	[1, 0, 1, −2, 0, 0, 0, 0]
Momentum	mv	$l_r m t^{-1}$	[1, 0, 1, −1, 0, 0, 0, 0]
Impulse	$\int F dt$	$l_r m t^{-1}$	[1, 0, 1, −1, 0, 0, 0, 0]
Torque	$\mathcal{T} = Fr$	$l_r l_t m t^{-2}$	[1, 1, 1, −2, 0, 0, 0, 0]
Energy	$U = \int F ds$	$l_r^2 m t^{-2}$**	[2, 0, 1, −2, 0, 0, 0, 0]
Power	$P = dU/dt$	$l_r^2 m t^{-3}$**	[2, 0, 1, −3, 0, 0, 0, 0]
Mass	m	m	[0, 0, 1, 0, 0, 0, 0, 0]
Moment of inertia	$\mathcal{I} = mr^2$	$l_r^2 m$	[2, 0, 1, 0, 0, 0, 0, 0]
Dissipation coefficient	$B = F/v$	$m t^{-1}$	[0, 0, 1, −1, 0, 0, 0, 0]
Stiffness	$K = F/s$	$m t^{-2}$	[0, 0, 1, −2, 0, 0, 0, 0]
Efficiency	$\eta = P_o/P_i$	1	[0]
Elasticity			
Stress, normal	$T_n = F_r/A$	$l_r l_t^{-2} m t^{-2}$	[1, −2, 1, −2, 0, 0, 0, 0]
shear	$T_s = F_t/A$	$l_t^{-1} m t^{-2}$	[0, −1, 0, −2, 0, 0, 0, 0]
Strain, normal	$S_n = \Delta s/s$	1	[0]
shear	$S_s = \Delta s_t/s_r$	$l_r^{-1} l_t$	[−1, 1, 0, 0, 0, 0, 0, 0]
Elastic moduli	T/S	$l_r l_t^{-2} m t^{-2}$	[1, −2, 1, −2, 0, 0, 0, 0]
Hydrodynamics			
Pressure	$p = F_r/A$	$l_r l_t^{-2} m t^{-2}$	[1, −2, 1, −2, 0, 0, 0, 0]
Velocity potential	$\varphi = \int v ds$	$l_r^2 t^{-1}$	[2, 0, 0, −1, 0, 0, 0, 0]
Density	$\delta = m/\mathcal{V}$	$l_r^{-1} l_t^{-2} m$	[−1, −2, 1, 0, 0, 0, 0, 0]
Viscosity	$\eta = \dfrac{T_s}{\Delta v_t / s_r}$	$l_r l_t^{-2} m t^{-1}$	[1, −2, 1, −1, 0, 0, 0, 0]
Surface tension	$\mathcal{L} = F_r/s_t$	$l_r l_t^{-1} m t^{-2}$	[1, −1, 1, −2, 0, 0, 0, 0]
Heat			
Temperature	T	T	[0, 0, 0, 0, 1, 0, 0, 0]
Thermal energy (heat)	U	$l_r^2 m t^{-2}$	[2, 0, 1, −2, 0, 0, 0, 0]
Thermal power	P	$l_r^2 m t^{-3}$	[2, 0, 1, −3, 0, 0, 0, 0]

* Or $l_t m t^{-2}$, depending on direction of force.
** Or $l_t^2 m t^{-2}$ and $l_t^2 m t^{-3}$, depending on direction of force.

TABLE IV.—*Continued.*

Name	Defining Equation	Idonic Formula	Base Vector $[l_r, l_i, m, t, T, \lambda, Q, F]$
Entropy	$\mathbb{S} = \int \frac{du}{T}$	$l_r^2 m t^{-2} T^{-1}$	[2, 0, 1, −2, −1, 0, 0, 0]
Power per unit volume	$\mathfrak{p} = P/\mathcal{U}$	$l_r l_i^{-2} m t^{-3}$	[1, −2, 1, −3, 0, 0, 0, 0]
Thermal conductivity	$k = \frac{P s_r}{A \Delta T}$	$l_r^3 l_i^{-2} m t^{-3} T^{-1}$	[3, −2, 1, −3, −1, 0, 0, 0]
Thermitivity (sp. heat)	$c = \frac{1}{m} \frac{\Delta U}{\Delta T}$	$l_r^2 t^{-2} T^{-1}$	[2, 0, 0, −2, −1, 0, 0, 0]
Diffusivity	$h^2 = k/\delta c$	$l_r^2 t^{-1}$	[2, 0, 0, −1, 0, 0, 0, 0]
Thalpance ("emissivity")	$\varrho = \frac{P}{A \Delta T}$	$l_r^2 l_i^{-2} m t^{-3} T^{-1}$	[2, −2, 1, −3, −1, 0, 0, 0]
Coefficient of linear expansion	$\alpha = \Delta l / l \Delta T$	T^{-1}	[0, 0, 0, 0, −1, 0, 0, 0]
Thermoelectric coefficient	$= \Delta V / \Delta T$	$l_i^2 m t^{-2} T^{-1} Q^{-1}$	[0, 2, 1, −2, −1, 0, −1, 0]
Temperature coefficient of resistance	$\alpha = \Delta R / (R \Delta T)$	T^{-1}	[0, 0, 0, 0, −1, 0, 0, 0]
Electrostatics			
Charge	Q	Q	[0, 0, 0, 0, 0, 0, 1, 0]
Charge per unit length	$\lambda = Q/s_i$	$Q l_i^{-1}$	[0, −1, 0, 0, 0, 0, 1, 0]
Charge per unit area	$\sigma = Q/A$	$Q l_i^{-2}$	[0, −2, 0, 0, 0, 0, 1, 0]
Charge per unit volume	$\rho = Q/\mathcal{U}$	$Q l_r^{-1} l_i^{-2}$	[−1, −2, 0, 0, 0, 0, 1, 0]
Electric field strength	$E = F/Q$	$l_i m t^{-2} Q^{-1}$	[0, 1, 1, −2, 0, 0, 1, 0]
Electric potential (and potential difference)	$V = \int E ds$	$l_i^2 m t^{-2} Q^{-1}$	[0, 2, 1, −2, 0, 0, 1, 0]
Electric flux density	$D = Q/A_{rt}$	$Q l_i^{-1} l_r^{-1}$	[−1, −1, 0, 0, 0, 0, 1, 0]
Electric flux	$\Psi = \int D dt$	Q	[0, 0, 0, 0, 0, 0, 1, 0]
Permittivity	$\epsilon = D/E$	$l_r^{-1} l_i^{-2} m^{-1} t^2 Q^2$	[−1, −2, 1, 2, 0, 0, 2, 0]
Capacitance	$C = Q/V$	$l_i^{-2} m^{-1} t^2 Q^2$	[0, −2, −1, 2, 0, 0, 2, 0]
Electric Current			
Current	$I = dQ/dt$	$Q t^{-1}$	[0, 0, 0, −1, 0, 0, 1, 0]
Current density	$J = dI/dA$	$l_r^{-1} l_i^{-1} Q t^{-1}$	[−1, −1, 0, −1, 0, 0, 1, 0]
Resistivity	$\mathfrak{R} = E/J$	$l_r l_i^3 m t^{-1} Q^{-2}$	[1, 2, 1, −1, 0, 0, −2, 0]
Resistance	$R = V/I$	$l_i^2 m t^{-1} Q^{-2}$	[0, 2, 1, −1, 0, 0, −2, 0]
Magnetism			
Magnetic flux density	$B = F_i / I l_i$	$m t^{-1} Q^{-1}$	[0, 0, 1, −1, 0, 0, −1, 0]
Magnetic flux	$\Phi = \int B da$	$l_i^2 m t^{-1} Q^{-1}$	[0, 2, 1, −1, 0, 0, −1, 0]
Magnetic field strength	$H = NI/l_r$	$l_r^{-1} t^{-1} Q$	[−1, 0, 0, −1, 0, 0, 1, 0]
Permeability	$\mu = B/H$	$l_r m Q^{-2}$	[1, 0, 1, 0, 0, 0, −2, 0]
Magnetomotive force	$\mathfrak{F} = NI$	$Q t^{-1}$	[0, 0, 0, −1, 0, 0, 1, 0]
Inductance	$L = N\Phi/I$	$l_i^2 m Q^{-2}$	[0, 2, 1, 0, 0, 0, −2, 0]
Poynting vector	$\mathbf{D}_r = \mathbf{E} \times \mathbf{H}$	$l_r^{-1} l_i m t^{-3}$	[−1, 1, 1, −3, 0, 0, 0, 0]
Radiometry			
Radiant pharos (radiant power)	$F_r = P$	$l_r^2 m t^{-3}$	[2, 0, 1, −3, 0, 0, 0, 0]
Radiant pharosage	$D_r = F_r/A$	$l_r^2 l_i^{-2} m t^{-3}$	[2, −2, 1, −3, 0, 0, 0, 0]
Radiant pharosum	$\mathfrak{p}_r = F_r/\mathcal{U}$	$l_r l_i^{-2} m t^{-3}$	[1, −2, 1, −3, 0, 0, 0, 0]
Radiant phos (energy)	$Q_r = U$	$l_r^2 m t^{-2}$	[2, 0, 1, −2, 0, 0, 0, 0]

TABLE IV.—*Continued.*

Name	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, m, t, T, \lambda, Q, F]$
Radiant phosage	$W_r = Q_r/A$	$l_r^2 l_t^{-2} mt^{-2}$	[2, −2, 1, −2, 0, 0, 0, 0]
Radiant phosum	$u_r = Q_r/\mathcal{U}$	$l_r l_t^{-2} mt^{-2}$	[1, −2, 1, −2, 0, 0, 0, 0]
Radiant helios	$H_r = \pi \frac{dD_r}{d\Omega}$	$l_r^4 l_t^{-4} mt^{-2}$	[4, −4, 1, −3, 0, 0, 0, 0]
Radiant heliosent	$G_r = H_r/s$	$l_r^3 l_t^{-4} mt^{-2}$	[3, −4, 1, −3, 0, 0, 0, 0]
Phengosage	$J(\lambda) = dD_r/d\lambda$	$l_r^3 l_t^{-2} mt^{-2} \lambda^{-1}$	[2, −2, 1, −3, 0, −1, 0, 0]
Photometry			
Luminous pharos (luminous flux)	F_l	F	[0, 0, 0, 0, 0, 0, 0, 1]
Luminous pharosage	$D_l = F_l/A$	$F l_t^{-2}$	[0, −2, 0, 0, 0, 0, 0, 1]
Luminous pharosum	$p_l = F_l/\mathcal{U}$	$F l_r^{-1} l_t^{-2}$	[−1, −2, 0, 0, 0, 0, 0, 1]
Luminous phos ("light")	$Q_l = F_l t$	Ft	[0, 0, 0, 1, 0, 0, 0, 1]
Luminous phosage	$W_l = Q_l/A$	$F l l_t^{-2}$	[0, −2, 0, 1, 0, 0, 0, 1]
Luminous phosum	$u_l = Q_l/\mathcal{U}$	$F l l_r^{-1} l_t^{-2}$	[−1, −2, 0, 1, 0, 0, 0, 1]
Luminous helios	$H_l = \pi \frac{dD_l}{d\Omega}$	$F l_r^3 l_t^{-4}$	[2, −4, 0, 0, 0, 0, 0, 1]
Luminous heliosent	$G_l = H_l/s$	$F l_r l_t^{-4}$	[1, −4, 0, 0, 0, 0, 0, 1]
Lamprosity ("visibility")	$\mathcal{G} = D_l/D_r$	$F l_r^{-2} m^{-1} t^3$	[0, −2, −1, 3, 0, 0, 0, 1]

For example, *force* is generally considered to be in the l_r-direction in accordance with Rule 1, the idonic formula being written $l_r mt^{-2}$. But if the force acts about an axis, it must be $l_t mt^{-2}$ in accordance with Rule 3. Thus *torque* is expressed idonically as $l_r l_t mt^{-2}$, while energy or "work" is $l_r^2 mt^{-2}$. A similar distinction is made between *normal stress* and *shear stress*. Normal stress is taken in the l_r-direction in accordance with Rules 1 and 2, but a shear force is tangent to an area and is therefore called $l_t mt^{-2}$ by Rule 2.

The most troublesome part of classical physics is electromagnetism, where the involved spacial relationships require thoughtful consideration if a consistent idonic specification is to be obtained. If electrostatics only is under consideration, the **E**-vector may be taken in the l_r-direction (Rule 1). But if all of electromagnetism is to form a consistent whole, the scheme of Fig. 2 seem preferable. Here the **B**-vector is taken in the l_r-direction, which gives l_t^2 as the area needed in obtaining total magnetic flux. Then **E** and **J** are along one l_t-axis, while the force **F** on a current-carrying conductor is along the other l_t-axis. This arbitrary arrangement gives the particular designation symbols given in Table IV for electromagnetism. Needless to say, the principle of idonic relativity allows many other specifications of equal validity.

As an indication that the designation symbols apply to other equations beside the particular ones employed in defining the concepts, we consider three other equations of electromagnetism. The first equation is concerned with energy stored in an electric field:

$$u = \tfrac{1}{2}\epsilon E^2. \qquad (6)$$

The energy per unit volume u is related to the force in the **E**-direction (l_t-direction) and is therefore

$$\frac{ml_t^2t^{-2}}{l_rl_t^3} = \mathbf{m}l_r^{-1}\mathbf{t}^{-2}.$$

Now consider the right side of Eq. 6. From Table IV,

$$(l_r^{-1}l_t^{-2}m^{-1}t^2Q^2)(l_t^2m^2t^{-2}Q^{-2}) = \mathbf{m}l_r^{-1}\mathbf{t}^{-2}.$$

Thus the designation symbols of Table IV satisfy Eq. (6).

As a second illustration, take the equation

$$\epsilon\mu = 1/c^2 \tag{7}$$

containing the velocity c of an electromagnetic wave. Since the wave is propagated in the l_t-direction (Fig. 2),

$$\frac{1}{c^2} = \mathbf{l}_t^{-2}\mathbf{t}^2.$$

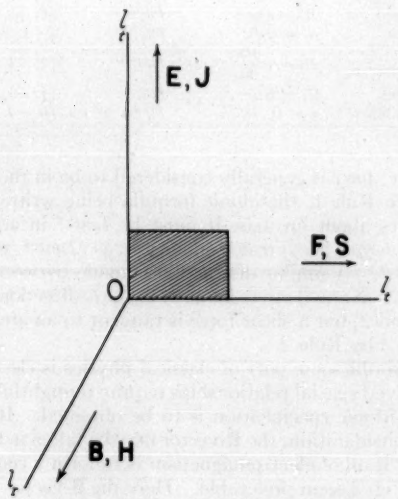

FIG. 2. Direction vectors for electromagnetic vectors.

The left side of Eq. 2 gives

$$(l_r^{-1}l_t^{-2}m^{-1}t^2Q^2)(l_rmQ^{-2}) = \mathbf{l}_t^{-2}\mathbf{t}^2.$$

The third example deals with the Poynting vector **S**:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}. \tag{8}$$

Since **S** is the power per unit area propagated horizontally (Fig. 2) in the l_t-direction, its idonic specification is

$$\frac{ml_t^2t^{-3}}{l_rl_t} = \mathbf{l}_r^{-1}\mathbf{l}_t\mathbf{m}\mathbf{t}^{-3}.$$

For the right side of Eq. 8, we have

$$(l_r m t^{-2} Q^{-1})(l_r^{-1} t^{-1} Q) = \mathbf{m} l_r^{-1} l_t t^{-3}$$

which again checks.

TABLE V.—*Ambiguities in Designations of Table IV.*
(l_r and l_t taken as basic lengths.)

Concepts	Base Vector $[l_r, l_t, m, t, T, \lambda, Q, F]$
Momentum, impulse	[1, 0, 1, −1, 0, 0, 0, 0]
Angle, shear strain	[−1, 1, 0, 0, 0, 0, 0, 0]
Normal stress, pressure, elastic moduli, radiant phosum	[1, −2, 1, −2, 0, 0, 0, 0]
Coefficient of linear expansion, temperature coefficient of resistance	[0, 0, 0, 0, −1, 0, 0, 0]
Velocity potential, diffusivity	[2, 0, 0, −1, 0, 0, 0, 0]
Charge, electric flux	[0, 0, 0, 0, 0, 0, 1, 0]
Current, mmf.	[0, 0, 0, −1, 0, 0, 1, 0]

TABLE VI.—*Kinematics.*

Concept	Symbol	Base Vector $[l_r, l_t, t]$
Distance	s	[1, 0, 0]
Area	A	[0, 2, 0]
Volume	$\mho$	[1, 2, 0]
Angle	θ	[−1, 1, 0]
Solid angle	Ω	[−2, 2, 0]
Time	t	[0, 0, 1]
Velocity	v	[1, 0, −1]
Acceleration	a	[1, 0, −2]
Angular velocity	ω	[−1, 1, −1]
Angular acceleration	α	[−1, 1, −2]
Frequency	f	[0, 0, −1]

TABLE VII.—*Dynamics.*

Concept	Symbol	Base Vector $[l_r, l_t, m, t]$
Force	F	[1, 0, 1, −2]
Momentum	mv	[1, 0, 1, −1]
Impulse	$\int F dt$	[1, 0, 1, −1]
Torque	$\mathcal{T}$	[1, 1, 1, −2]
Energy	U	[2, 0, 1, −2]
Power	P	[2, 0, 1, −3]
Mass	m	[0, 0, 1, 0]
Moment of inertia	$\mathcal{I}$	[2, 0, 1, 0]
Dissipation coefficient	B	[0, 0, 1, −1]
Stiffness	K	[0, 0, 1, −2]
Efficiency	η	[0]

That l_r and l_t provide a good system (9) for designating the concepts of physics is shown by Table V. Most of the ambiguities of the other systems (Tables II and III) have been eliminated by the use of two length concepts. Most of the remaining duplications are evidently redundancies in the formulation of physical concepts rather than errors in the idonic method. For example, shear strain is actually an angle, and this fact is evidenced by the identity of designation symbols for the two concepts. Similarly, there is no way to distinguish idonically between *electric charge* and *electric flux* or between *current* and *magnetomotive force*. On the other hand, there are a few cases, such as *pressure* and *radiant energy per unit volume* or *velocity potential* and *diffusivity*, where the concepts are physically distinct but the designation symbols are identical. One may regard these few cases as defects of the $l_r l_t$-system, though it is doubtful if they could cause any inconvenience.

Tables I and IV indicate that the designation symbols are somewhat cumbersome when based on the primary concepts needed for all of

TABLE VIII.—*Heat.*

Concept	Symbol	Base Vector $[l_r, l_t, t, T, P]$
Temperature	T	[0, 0, 0, 1, 0]
Thermal energy (heat)	U	[0, 0, 1, 0, 1]
Thermal power	P	[0, 0, 0, 0, 1]
Entropy	$\mathfrak{S}$	[0, 0, 1, −1, 1]
Power per unit volume	$\mathfrak{p}$	[−1, −2, 0, 0, 1]
Thermal conductivity	k	[1, −2, 0, −1, 1]
Thermitivity	c	[2, 0, −2, −1, 0]
Diffusivity	k^2	[2, 0, −1, 0, 0]
Thalpance	$\mathfrak{d}$	[0, −2, 0, −1, 1]
Coefficient of linear expansion	α	[0, 0, 0, −1, 0]

physics. It should be realized, however, that in any given problem the complete set of concepts is ordinarily not required. In kinematics, for instance, only l_r, l_t, and t are required and idon space is three-dimensional. The designation symbols are listed in Table VI. For dynamics, an additional primary concept (such as *mass*) is introduced; but the base vectors are still fairly simple, as shown in Table VII.

When a problem deals exclusively with heat, there is no need of expressing the designation symbols in terms of mechanical quantities. As shown by Table VIII, the base vectors are expressed very simply in terms of l_r, l_t, t, T, and P. Table IX lists the concepts of electromagnetism, expressed in terms of l_r, l_t, t, I, and P. For electromagnetic problems, this set of primary concepts is often found to be most convenient, though other sets (such as l_r, l_t, t, Q, U) are also good. In radiometry, the use of P rather than m as a primary concept exhibits

TABLE IX.—*Electricity and Magnetism.*

Concept	Symbol	Base Vector $[l_r, l_t, t, I, P]$
Electrostatics		
Charge	Q	[0, 0, 1, 1, 0]
Charge per unit length	λ	[0, −1, 1, 1, 0]
Charge per unit area	σ	[0, −2, 1, 1, 0]
Charge per unit volume	ρ	[−1, −2, 1, 1, 0]
Electric field strength	E	[0, −1, 0, −1, 1]
Electric potential	V	[0, 0, 0, −1, 1]
Electric flux density	D	[−1, −1, 1, 1, 0]
Permittivity	ϵ	[−1, 0, 1, 2, −1]
Capacitance	C	[0, 0, 1, 2, −1]
Electric Current		
Current	I	[0, 0, 0, 1, 0]
Current density	J	[−1, −1, 0, 1, 0]
Resistivity	$\mathfrak{R}$	[1, 0, 0, −2, 1]
Resistance	R	[0, 0, 0, −2, 1]
Magnetism		
Magnetic flux density	B	[0, −2, 1, −1, 1]
Magnetic flux	Φ	[0, 0, 1, −1, 1]
Magnetic field strength	H	[−1, 0, 0, 1, 0]
Permeability	μ	[1, −2, 1, −2, 1]
Mmf.	$\mathfrak{F}$	[0, 0, 0, 1, 0]
Inductance	L	[0, 0, 1, −2, 1]
Poynting vector	D_r	[−1, −1, 0, 0, 1]

TABLE X.—*Radiometry and Photometry.*

Concept	Symbol	Base Vector $[l_r, l_t, t, \lambda, P, F]$
Radiometry		
Radiant pharos (radiant power)	F_r	[0, 0, 0, 0, 1, 0]
Radiant pharosage	D_r	[0, −2, 0, 0, 1, 0]
Radiant pharosum	p_r	[−1, −2, 0, 0, 1, 0]
Radiant phos (radiant energy)	Q_r	[0, 0, 1, 0, 1, 0]
Radiant phosage	W_r	[0, −2, 1, 0, 1, 0]
Radiant phosum	u_r	[−1, −2, 1, 0, 1, 0]
Radiant helios	H_r	[2, −4, 0, 0, 1, 0]
Radiant heliosent	G_r	[1, −4, 0, 0, 1, 0]
Phengosage	$J(\lambda)$	[0, −2, 0, −1, 1, 0]
Photometry		
Luminous pharos (luminous flux)	F_l	[0, 0, 0, 0, 0, 1]
Luminous pharosage	D_l	[0, −2, 0, 0, 0, 1]
Luminous pharosum	p_l	[−1, −2, 0, 0, 0, 1]
Luminous phos ("light")	Q_l	[0, 0, 1, 0, 0, 1]
Luminous phosage	W_l	[0, −2, 1, 0, 0, 1]
Luminous phosum	u_l	[−1, −2, 1, 0, 0, 1]
Luminous helios	H_l	[2, −4, 0, 0, 0, 1]
Luminous heliosent	G_l	[1, −4, 0, 0, 0, 1]
Lamprosity ("visibility")	$\mathfrak{y}$	[0, 0, 0, 0, −1, 1]

the parallelism between the radiometric and photometric concepts (Table X). Tables VI to X are examples which show that designation symbols are not unduly complicated in the l,l_r-system and suggest some of the variety that is allowable under the principle of idonic relativity.

6. OTHER USES OF DESIGNATION SYMBOLS

Undoubtedly the principal value of designation symbols ("dimensions") is that they give a shorthand specification of concepts and show at a glance the relations among concepts. There are other uses, however, five of which were listed previously (3). Change of *units* is easily effected in the l,l_r-system by condensing the two l's into one:

$$l_r = l_t = l$$

and proceeding as in the *lmt* system. This is based on the usual assumption that the same unit of length is employed throughout. Note, however, that the l,l_r-system allows the calculation of change of units even when lengths are measured in different units in different directions (volumes in *acre feet*, for instance, or angles in *degrees*). Other uses of designation symbols are handled in the new system just as in the *lmt*-system. To demonstrate this fact, we give two illustrations.

The first example deals with the idonic checking of equations. Physicists and engineers frequently substitute idonic expressions on both sides of an equation to determine if it is "dimensionally" homogeneous. Homogeneity is a necessary but not a sufficient condition for the validity of the equation. If such a check is obtained with *lmt*, there is still considerable chance that the assumed equation is wrong. When additional fundamental concepts are introduced into the idonic system, the margin of error is reduced, though the method never insures validity.

Suppose that a physicist, accustomed to use Gaussian units and to omit ϵ_0 and μ_0, is trying to remember how to write the expression for the capacitance of a parallel-plate capacitor. He knows that C is directly proportional to the area A of the plates, inversely proportional to their separation l, and depends on the permittivity ϵ of the medium. How does ϵ enter the equation? From Table IX and Fig. 2, he obtains the following base bectors:

For C,	$[0, 0, 1, 2, -1]$
For A,	$[1, 1, 0, 0, 0]$
For l,	$[0, 1, 0, 0, 0]$
For ϵ,	$[-1, 0, 1, 2, -1]$.

Thus,

$$[0, 0, 1, 2, -1] = [1, 1, 0, 0, 0] - [0, 1, 0, 0, 0] \pm [-1, 0, 1, 2, -1].$$

Evidently, homogeneity is obtained with the + sign, so the equation

must be

$$C = \frac{\epsilon A}{l} \text{ (possible idonic scalar).} \tag{9}$$

Try the same problem with the customary *lmt* system. From Table I,

$$[1, 0, 0] = [2, 0, 0] - [1, 0, 0] \pm [0],$$

which tells absolutely nothing about the position of ϵ in the capacitance equation.

As a second illustration, consider the derivation of Child's relation on the idonic basis. Assume two large parallel electrodes, one of which emits electrons. Evidently, the pertinent concepts are the current density J (equilibrium conditions), the potential difference V between electrodes, charge Q of an electron, mass m of an electron, distance l between electrodes, and permittivity ϵ. The associated base vectors are obtained from Table IX, giving

$$\begin{aligned}
\{J\} &= J[-1, -1, 0, 1, 0] \\
\{V\} &= V[0, 0, 0, -1, 1] \\
\{Q\} &= Q[0, 0, 1, 1, 0] \\
\{m\} &= m[0, -2, 3, 0, 1] \\
\{l\} &= l[0, 1, 0, 0, 0] \\
\{\epsilon\} &= \epsilon[-1, 0, 1, 2, -1].
\end{aligned}$$

Let

$$\{J\} = \{V\}^{\alpha}\{Q\}^{\beta}\{m\}^{\gamma}\{l\}^{\delta}\{\epsilon\}^{\mu} \cdot \text{const.},$$

where $\alpha, \beta, \gamma, \delta, \mu$ are constants that are to be determined. Substitution of the foregoing idonic equations gives $\alpha = \frac{1}{2}, \beta = \frac{1}{2}, \gamma = -\frac{1}{2}, \delta = -2$, $\mu = 1$. Thus the equation for current density limited by space charge must be

$$J = \text{Const.} \sqrt{Q/m}\, \frac{\epsilon V^{\frac{1}{2}}}{l^2} \tag{10}$$

which is the usual form of Child's equation.

A less satisfactory result is obtained if a single fundamental length is employed instead of l_r and l_t. From Table I for base vectors $[l, m, t, Q]$,

$$\begin{aligned}
\{J\} &= J[-2, 0, -1, 1] \\
\{V\} &= V[2, 1, -2, -1] \\
\{Q\} &= Q[0, 0, 0, 1] \\
\{m\} &= m[0, 1, 0, 0] \\
\{l\} &= l[1, 0, 0, 0] \\
\{\epsilon\} &= \epsilon[-3, -1, 2, 2].
\end{aligned}$$

Using the same process as before, one finds that the equation must have

the form,

$$J = \text{Const.} \sqrt{\frac{V}{m}} \frac{Q^1}{l^3} \cdot F\left(\frac{Vl\epsilon}{Q}\right). \tag{11}$$

By proper choice of the unknown function F, we can again obtain Child's equation. But there is nothing in this second solution to indicate the form of F, and thus the result is much less satisfastory than that obtained with l_r and l_t.

7. A VECTOR SYSTEM

The previous sections have shown that there is a distinct advantage in the use of two fundamental lengths l_r and l_t in place of the usual l-"dimension." We now consider an alternative system in which the distinction between vectors and scalars is maintained in the idonic specification of physical concepts. It will be recalled that in the previous systems, the vector or scalar nature of the physical quantities is ignored and vector and scalar products are considered to be identical. In some cases, however, the additional information, obtained by discriminating between vectors and scalars in physical space, may be helpful.

The fundamental lengths l_r and l_t are again employed, but with an additional length l which is used where direction is not significant. The three rules of Section 5 are retained and a fourth rule is added:

(4) The defining equations of physics, used in determining the idonic specification, are written in vector form. The distinction between vector and scalar products is maintained, in accordance with the following equations:

$$\begin{aligned} l_t \times l_t &= l_r l \\ l_r \times l_t = l_t \times l_r &= l_t l, \\ l_t \cdot l_t &= l^2. \end{aligned}$$

These product rules are like those of ordinary vector algebra. The only difference is that the distinction between positive and negative signs in vector products is dropped, and hence the vector product becomes commutative.

Table XI lists the principal concepts of physics and their idonic specification in the vector system. *Distance* in general is considered as a vector, and (in accordance with Rule 1) it is in the l_r-direction. In accordance with the usual convention of vector analysis, *area* is taken as a vector that is perpendicular to the element of surface under consideration. *Volume* may be defined as $l_r l \cdot l_r$, which according to Rule 4 is equal to the scalar quantity l^3. *Torque* is

$$\mathbf{T} = \mathbf{s}_r \times \mathbf{F}_t$$

so its designation symbol is

$$l_r \times l_t m t^{-2} = l_t l m t^{-2},$$

which shows that torque is a vector quantity in the direction l_t.

TABLE XI.—*A Vectorial Derivation of Designation Symbols.*
(l_r, l_t, l taken as basic lengths.)

Concept	Symbol	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, l, m, t, T, \lambda, Q, F]$
Geometry				
Distance	$\mathbf{s}$	$\mathbf{s}$	l_r	[1, 0, 0, 0, 0, 0, 0, 0, 0]
Area	$\mathbf{A}$	$\mathbf{A} = \mathbf{x} \times \mathbf{y}$	$l_r l$	[1, 0, 1, 0, 0, 0, 0, 0, 0]
Volume	$\mathcal{V}$	$\mathcal{V} = \mathbf{A} \cdot \mathbf{s}$	l^3	[0, 0, 3, 0, 0, 0, 0, 0, 0]
Angle	θ	$\lvert\theta\rvert = \lvert s_t\rvert / \lvert s_r\rvert$	$l_t l_r^{-1}$	[−1, 1, 0, 0, 0, 0, 0, 0, 0]
Solid angle	Ω	$\Omega = \mathbf{A}/(\mathbf{r} \cdot \mathbf{r})$	$l_r l^{-1}$	[1, 0, −1, 0, 0, 0, 0, 0, 0]
Kinematics				
Time	t	t	t	[0, 0, 0, 0, 1, 0, 0, 0, 0]
Velocity	$\mathbf{v}$	$\mathbf{v} = d\mathbf{s}/dt$	$l_r t^{-1}$	[1, 0, 0, 0, −1, 0, 0, 0, 0]
Acceleration	$\mathbf{a}$	$\mathbf{a} = d\mathbf{v}/dt$	$l_r t^{-2}$	[1, 0, 0, 0, −2, 0, 0, 0, 0]
Angular velocity	ω	$\omega = d\lvert\theta\rvert/dt$	$l_t l_r^{-1} t^{-1}$	[−1, 1, 0, 0, −1, 0, 0, 0, 0]
Angular acceleration	α	$\lvert\alpha\rvert = \lvert d\omega/dt\rvert$	$l_t l_r^{-1} t^{-2}$	[−1, 1, 0, 0, −2, 0, 0, 0, 0]
Frequency	f	$f = 1/t$	t^{-1}	[0, 0, 0, 0, −1,0, 0, 0, 0]
Dynamics				
Force	$\mathbf{F}$	$\mathbf{F} = m\mathbf{a}$	$l_r m t^{-2}$*	[1, 0, 0, 1, −2, 0, 0, 0, 0]
Momentum		$m\mathbf{v}$	$l_r m t^{-1}$	[1, 0, 0, 1, −1, 0, 0, 0, 0]
Impulse		$\int \mathbf{F} dt$	$l_r m t^{-1}$	[1, 0, 0, 1, −1, 0, 0, 0, 0]
Torque	$\mathfrak{T}$	$\mathfrak{T} = \mathbf{r} \times \mathbf{F}_t$	$l_t l m t^{-2}$	[0, 1, 1, 1, −2, 0, 0, 0, 0]
Energy	U	$U = \int \mathbf{F} \cdot d\mathbf{s}$	$l^2 m t^{-2}$	[0, 0, 2, 1, −2, 0, 0, 0, 0]
Power	P	$P = dU/dt$	$l^2 m t^{-3}$	[0, 0, 2, 1, −3, 0, 0, 0, 0]
Mass	m		m	[0, 0, 0, 1, 0, 0, 0, 0, 0]
Moment of inertia	$\mathcal{I}$	$\mathcal{I} = \int \mathbf{r} \cdot \mathbf{r} dm$	$l^2 m$	[0, 0, 2, 1, 0, 0, 0, 0, 0]
Dissipation coefficient	B	$\mathbf{F} = B\mathbf{v}$	$m t^{-1}$	[0, 0, 0, 1, −1, 0, 0, 0, 0]
Stiffness	K	$\mathbf{F} = K\mathbf{s}$	$m t^{-2}$	[0, 0, 0, 1, −2, 0, 0, 0, 0]
Efficiency*	η	$\eta = P_o/P_i$	1	[0]
Elasticity				
Stress, normal	T_n	$\lvert T_n\rvert = \lvert F_r\rvert / \lvert A_r\rvert$	$l^{-1} m t^{-2}$	[0, 0, −1, 1, −2, 0, 0, 0, 0]
shear	T_s	$\lvert T_s\rvert = \lvert F_t\rvert / \lvert A_r\rvert$	$l_t l_r^{-1} l^{-1} m t^{-2}$	[−1, 1, −1, 1, −2, 0, 0, 0, 0]
Strain, normal	S_n	$\lvert S_n\rvert = \lvert \Delta s_r\rvert / \lvert s_r\rvert$	1	[0]
shear	S_t	$\lvert S_t\rvert = \lvert \Delta s_t\rvert / \lvert s_r\rvert$	$l_t l_r^{-1}$	[−1, 1, 0, 0, 0, 0, 0, 0, 0]
Elastic moduli	c	$\lvert c\rvert = \lvert T\rvert / \lvert S\rvert$	$l^{-1} m t^{-2}$	[0, 0, −1, 1, −2, 0, 0, 0, 0]
Hydrodynamics				
Pressure	p	$p = \lvert F_r\rvert / \lvert A_r\rvert$	$l^{-1} m t^{-2}$	[0, 0, −1, 1, −2, 0, 0, 0, 0]
Velocity potential	φ	$\varphi = \int \mathbf{v} \cdot d\mathbf{s}$	$l^2 t^{-1}$	[0, 0, 2, 0, −1, 0, 0, 0, 0]
Density	δ	$\delta = m/\mathcal{V}$	$m l^{-3}$	[0, 0, −3, 1, 0, 0, 0, 0, 0]
Viscosity	η	$\lvert T_s\rvert = \eta \lvert \Delta v_t\rvert / \lvert s_r\rvert$	$l^{-1} m t^{-1}$	[0, 0, −1, 1, −1, 0, 0, 0, 0]
Surface tension	$\mathcal{L}$	$\lvert F_r\rvert = \mathcal{L} \lvert s_t\rvert$	$l_r l_t^{-1} m t^{-2}$	[1, −1, 0, 1, −2, 0, 0, 0, 0]
Heat				
Temperature	T		T	[0, 0, 0, 0, 0, 1, 0, 0, 0]
Thermal energy (heat)	U		$l^2 m t^{-2}$	[0, 0, 2, 1, −2, 0, 0, 0, 0]
Thermal power	P		$l^2 m t^{-3}$	[0, 0, 2, 1, −3, 0, 0, 0, 0]
Entropy	$\mathcal{S}$	$\mathcal{S} = \int dU/T$	$l^2 m t^{-2} T^{-1}$	[0, 0, 1, 1, −2, −1, 0, 0, 0]
Power per unit volume	$\mathfrak{p}$	$\mathfrak{p} = P/\mathcal{V}$	$l^{-1} m t^{-3}$	[0, 0, −1, 1, −3, 0, 0, 0, 0]

TABLE XI.—*Continued.*

Concept	Symbol	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, l, m, t, T, \lambda, Q, F]$
Thermal conductivity	k	$kA\Delta T = P\mathbf{s}_r$	$lmt^{-3}T^{-1}$	[0, 0, 1, 1, −3, −1, 0, 0, 0]
Thermitivity (sp. heat)	c	$c = \frac{1}{m}\frac{\Delta U}{\Delta T}$	$l^2t^{-2}T^{-1}$	[0, 0, 2, 0, −2, 1, 0, 0, 0]
Diffusivity	h^2	$h^2 = \frac{k}{\delta c}$	l^2t^{-1}	[0, 0, 2, 0, −1, 0, 0, 0, 0]
Thalpance ("emissivity")	$\mathfrak{p}$	$\lvert\mathfrak{p}\rvert = \frac{P}{\lvert A\rvert\Delta T}$	$mt^{-3}T^{-1}$	[0, 0, 0, 1, −3, −1, 0, 0, 0]
Coefficient of linear expansion	α	$\Delta l = \alpha l\Delta T$	T^{-1}	[0, 0, 0, 0, 0, −1, 0, 0, 0]
Thermoelectric coefficient	$\mathfrak{z}$	$\Delta V = \mathfrak{z}\Delta T$	$l^2mt^{-3}T^{-1}Q^{-1}$	[0, 0, 2, 1, −2, −1, 0, −1, 0]
Temperature coefficient of resistance	α	$\Delta R = \alpha R\Delta T$	T^{-1}	[0, 0, 0, 0, 0, −1, 0, 0, 0]
Electrostatics				
Charge	Q		Q	[0, 0, 0, 0, 0, 0, 0, 1, 0]
Charge per unit length	λ	$\lvert\lambda\rvert = Q/\lvert s_r\rvert$	Ql^{-1}	[0, 0, −1, 0, 0, 0, 0, 1, 0]
Charge per unit area	σ	$\lvert\sigma\rvert = Q/\lvert A\rvert$	Ql^{-2}	[0, 0, −2, 0, 0, 0, 0, 1, 0]
Charge per unit volume	ρ	$\rho = Q/\mathcal{V}$	Ql^{-3}	[0, 0, −3, 0, 0, 0, 0, 1, 0]
Electric field strength	$\mathbf{E}$	$\mathbf{E} = \mathbf{F}/Q$	$l_rmt^{-3}Q^{-1}$**	[1, 0, 0, 1, −2, 0, 0, −1, 0]
Electric potential (and potential difference)	V	$V = \int\mathbf{E}\cdot d\mathbf{s}$	$l^2mt^{-3}Q^{-1}$	[0, 0, 2, 1, −2, 0, 0, −1, 0]
Electric flux density	$\mathbf{D}$	$Q = \int\mathbf{D}\cdot d\mathbf{A}$	$l_rl^{-3}Q$***	[1, 0, −3, 0, 0, 0, 0, 1, 0]
Electric flux	Ψ	$\Psi = \int\mathbf{D}\cdot d\mathbf{A}$	Q	[0, 0, 0, 0, 0, 0, 0, 1, 0]
Permittivity	ϵ	$\mathbf{D} = \epsilon\mathbf{E}$	$l^{-3}m^{-1}t^2Q^2$	[0, 0, −3, −1, 2, 0, 0, 2, 0]
Capacitance	C	$C = Q/V$	$l^{-2}m^{-1}t^2Q^2$	[0, 0, −2, −1, 2, 0, 0, 2, 0]
Electric Current				
Current	I	$I = dQ/dt$	Qt^{-1}	[0, 0, 0, 0, −1, 0, 0, 1, 0]
Current density	$\mathbf{J}$	$I = \int\mathbf{J}\cdot d\mathbf{A}$	$l_rl^{-2}t^{-1}Q$	[1, 0, −3, 0, −1, 0, 0, 1, 0]
Resistivity	$\mathfrak{R}$	$\mathbf{E} = \mathfrak{R}\mathbf{J}$	$l^3mt^{-1}Q^{-2}$	[0, 0, 3, 1, −1, 0, 0, −2, 0]
Resistance	R	$R = V/I$	$l^2mt^{-1}Q^{-2}$	[0, 0, 2, 1, −1, 0, 0, −2, 0]
Magnetism				
Magnetic flux density	$\mathbf{B}$	$\mathbf{F}_i = I(\mathbf{s}_i \times \mathbf{B})$	$l_rl^{-1}mt^{-1}Q^{-1}$	[1, 0, −1, 1, −1, 0, 0, −1, 0]
Magnetic flux	Φ	$\Phi = \int\mathbf{B}\cdot d\mathbf{A}$	$l^2mt^{-1}Q^{-1}$	[0, 0, 1, 1, −1, 0, 0, 1, 0]
Magnetic field strength	$\mathbf{H}$	$I = \int\mathbf{H}\cdot d\mathbf{s}$	$l_rl^{-2}t^{-1}Q$	[1, 0, −2, 0, −1, 0, 0, 1, 0]
Permeability	μ	$\mathbf{B} = \mu\mathbf{H}$	lmQ^{-2}	[0, 0, 1, 1, 0, 0, 0, −2, 0]
Mmf.	$\mathfrak{F}$	$\mathfrak{F} = NI$	Qt^{-1}	[0, 0, 0, 0, −1, 0, 0, 1, 0]
Vector potential	$\mathbf{A}$	$\mathbf{B} = \text{curl}\,\mathbf{A}$	$l_r^{-1}l^2mt^{-1}Q^{-1}$	[0, −1, 2, 1, −1, 0, 0, −1, 0]
Inductance	L	$L = N\Phi/I$	l^2mQ^{-2}	[0, 0, 2, 1, 0, 0, 0, −2, 0]
Poynting vector	$\mathbf{D}_r$	$\mathbf{D}_r = \mathbf{E}_i \times \mathbf{H}_i$	$l_rl^{-1}mt^{-3}$	[1, 0, −1, 1, −3, 0, 0, 0, 0]

TABLE XI.—*Continued.*

Concept	Symbol	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, l, m, t, T, \lambda, Q, F]$
Radiometry				
Radiant pharos (radiant power)	F_r	$F_r = P$	l^2mt^{-3}	[0, 0, 2, 1, −3, 0, 0, 0, 0]
Radiant pharosage	$\mathbf{D}_r$	$F_r = \int \mathbf{D}_r \cdot d\mathbf{A}$	$l_r l^{-1} mt^{-3}$	[1, 0, −1, 1, −3, 0, 0, 0, 0]
Radiant pharosum	$\mathfrak{p}_r$	$\mathfrak{p}_r = F_r/\mathfrak{U}$	$l^{-1}mt^{-3}$	[0, 0, −1, 1, −3, 0, 0, 0, 0]
Radiant phos (energy)	Q_r	$Q_r = U$	l^2mt^{-2}	[0, 0, 2, 1, −2, 0, 0, 0, 0]
Radiant phosage	$\mathbf{W}_r$	$Q_r = \int \mathbf{W}_r \cdot d\mathbf{A}$	$l_r l^{-1} mt^{-2}$	[−1, 0, 1, 1, −2, 0, 0, 0, 0]
Radiant phosum	u_r	$u_r = Q_r/\mathfrak{U}$	$l^{-1}mt^{-2}$	[0, 0, −1, 1, −2, 0, 0, 0, 0]
Radiant helios	H_r	$d\mathbf{D}_r = \frac{1}{\pi} H_r d\boldsymbol{\Omega}$	mt^{-3}	[0, 0, 0, 1, −3, 0, 0, 0, 0]
Radiant heliosent	$\mathbf{G}_r$	$H_r = \int \mathbf{G}_r \cdot d\mathbf{s}_r$	$l_r l^{-2} mt^{-3}$	[1, 0, −2, 1, −3, 0, 0, 0, 0]
Phengosage	$\mathbf{J}(\lambda)$	$\mathbf{D}_r = \int \mathbf{J}(\lambda) d\lambda$	$l_r l^{-1} mt^{-3} \lambda^{-1}$	[1, 0, −1, 1, −3, 0, −1, 0, 0]
Photometry				
Luminous pharos (luminous flux)	F_l		F	[0, 0, 0, 0, 0, 0, 0, 0, 1]
Luminous pharosage	D_l	$F_l = \int \mathbf{D}_l d\mathbf{A}$	$l_r l^{-2} F$	[1, 0, −3, 0, 0, 0, 0, 0, 1]
Luminous pharosum	P_l	$P_l = F_l/\mathfrak{U}$	$l^{-3}F$	[0, 0, −3, 0, 0, 0, 0, 0, 1]
Luminous phos ("light")	Q_l	$Q_l = \int F_l dt$	tF	[0, 0, 0, 0, 1, 0, 0, 0, 1]
Luminous phosage	$\mathbf{W}_l$	$Q_l = \int \mathbf{W}_l \cdot d\mathbf{A}$	$l_r l^{-2} tF$	[1, 0, −3, 0, 1, 0, 0, 0, 1]
Luminous phosum	u_l	$u_l = Q_l/\mathfrak{U}$	$l^{-3}tF$	[0, 0, −3, 0, 1, 0, 0, 0, 1]
Luminous helios	H_l	$d\mathbf{D}_l = \frac{1}{\pi} H_l d\boldsymbol{\Omega}$	$l^{-2}F$	[0, 0, −2, 0, 0, 0, 0, 0, 1]
Luminous heliosent	$\mathbf{G}_l$	$H_l = \int \mathbf{G}_l \cdot d\mathbf{s}_r$	$l_r l^{-4} F$	[1, 0, −4, 0, 0, 0, 0, 0, 1]
Lamprosity ("visibility")	$\mathfrak{y}$	$\mathfrak{y} = D_l/D_r$	$l^{-2}m^{-1}t^{-2}F$	[0, 0, −2, −1, −3, 0, 0, 0, 1]

* Or $l_r mt^{-2}$, depending on direction of force.
** Or $l_r mt^{-2}Q^{-1}$, depending on direction of **E**.
*** Or $l_t l^{-2}Q$, depending on direction of **E**.

As another example, consider the idonic specification of magnetic flux density. The defining equation is

$$\mathbf{F}_t = \mathbf{I}(\mathbf{s}_t \times \mathbf{B}),$$

where $\mathbf{s}_t$ is in the direction of the current. The only area that is significant when dealing with magnetic flux density is that perpendicular to **B**, so **B** is in the l_r-direction and **F** and **s** are in the l_t directions (Fig. 2). Substituting the idonic specification in the defining equation, one obtains

$$l_t mt^{-2} = (Qt^{-1})(l_t \times B),$$

which is satisfied by

$$B = l_r l^{-1} mt^{-1} Q^{-1}.$$

A word should be said about angle and solid angle. According to Rule 3, the tangential distance is called l_t and the radial distance l_r, so

the idonic specification of angle is $l_t l_r^{-1}$. Closely associated with this representation of angle is our treatment of stress and strain. An adequate treatment would require tensor analysis, but for many engineering purposes it is customary to consider only magnitudes. Normal stress is then specified idonically by

$$\frac{l_r m t^{-2}}{l_t l} = l^{-1} m t^{-2}.$$

Shear stress, on the other hand, is specified by $l_r m t^{-2}/l_t l = l_t l_r^{-1} l^{-1} m t^{-2}$. In this way, different designation symbols are given for the two stresses, just as different designation symbols are obtained for energy and torque.

Table XI presents the designation symbols for classical physics, in so far as physics can be described by ordinary vector analysis. The treatment is limited to euclidean space and isotropic media. A more general and more precise description of physical phenomena could be given in terms of tensors, and the designation symbols could be revised in accordance with this extension. Such a step, however, will not be taken in this paper.

The vector system eliminates ambiguities to essentially the same extent as the scalar $l_r l_t$-system did (Table V). The new system also provides a check on the vector character of a given equation. In many cases (as in the examples of Section 6) the physical problem does not need vectors. If this be true, the idonic treatment, on scalar $l_r l_t$ basis, is generally preferable. In other cases, the vector system is advantageous. Study of Table XI gives the following information:

(*a*) If a physical quantity is a scalar, neither l_r nor l_t enter the designation symbol.

(*b*) A vector quantity contains either l_r or l_t (but not both) in its idonic specification.

(*c*) The appearance of both l_r and l_t in a designation symbol indicates a quantity that is not adequately represented by ordinary vector analysis. Such a concept could be better represented by a covariant vector or by a bivector. These conclusions apply, of course, only to the particular representation given in Table XI, with the defining equations there listed. It should be emphasized again that "dimensions" are not absolute and that a system of designation symbols can be arranged in an infinite variety of ways to meet the requirements of the particular problems being considered.

8. SUMMARY

The paper describes an investigation of some possible "dimensional" systems for the designation of physical concepts. The usual system, based on the primary concepts of *length*, *mass*, and *time*, results in a large number of ambiguities (Table II) and is therefore unsatisfactory.

The introduction of an additional primary concept for heat and another for electromagnetism is an improvement, but many ambiguities still remain (Table III). The trouble is caused by our failure to distinguish between distances in different directions.

By introducing two fundamental lengths l_r and l_t (radial and tangential) in place of the usual l, we eliminate the former difficulty and obtain distinct designation symbols for the concepts of *energy* and *torque*, for *normal stress* and *shear stress*, for *pharosage* (radiant energy per unit area) and *helios* (brightness).

There are six types of practical application for idon theory ("dimensions"), all of which are handled as well (or better) by the $l_r l_t$-system as by the old method. Examples are given of the use of the new method in checking equations and in deriving new equations by idonic analysis. Tables of designation symbols are given for the principal concepts of physics (Table VI to X). Finally, an alternative system is outlined which employes the fundamental lengths l_r, l_t and l, and which discriminates between vectors and scalars in physical space (Table XI). This vector system is useful in certain problems, though in most cases the simpler $l_r l_t$-system is adequate.

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Automatically Vacuum-Cleaned Workbench (*Bottling*, No. 102).—A neat device has recently been placed on the market, and is of especial interest to bottlers operating their own repair shop, where buffing, grinding and other dust-creating processes have to be encountered. The bench is of re-inforced steel plate but on its working surface there is an 18-in. square rubber-covered grid communicating with a suction pump, dust filter and shaking gear, all housed in the interior of the bench. Hence the worker, especially if working with flexible tools, is completely protected from dangerous dusts, the surrounding atmosphere is kept clear, and if the metal or other dusts have any commercial value they can be recovered.

Fluid-Motor Valve Operator (*Rubber Age*, Vol. 65, No. 6).—A simplified fluid-motor operator for gate valves has been developed by the Crane Co., Chicago. This new design makes possible extensive use of motor-operated valves, since the actuating unit can be easily adapted to standard valves in stock or to valves already installed. This previously has been a virtual impossibility, for motor-operated valves have customarily been made up on special order to the requirements of the installation. The device can be operated by water, oil or compressed air or gases.

The motor unit, comprising the motor, gear box and yoke, is bolted to the valve bonnet; and the motor operates the valve stem through the gear box. The motor derives its power from five flexible diaphragms, mounted radially about an eccentric on the drive shaft. The operating fluid is admitted in rotational sequence to the five diaphragms, which actuate pistons that transmit the thrust to the eccentric through either of two entrance ports, controlling the direction of operation of the motor for opening and closing the valve.

When the valve gate reaches the end of its travel in either direction, the motor merely stalls, still under pressure. There can be no leakage, and shut-off devices are unnecessary. The valves can be operated by a wide range of liquids or gases at pressures from 40 to 300 psi.

Aerial-pickup System.—A "fishing reel" aerial-pickup system perfected by an Air Force officer in Panama makes it possible for a light plane to pick up messages and mail without descending to dangerously low altitudes. At 200 ft. the plane lets out, by gravity fall of a heavy lead sinker, a line to which is attached a 2-in. hook. This engages the mail bag or message container, suspended between two poles on the ground. When the "catch" is made, a certain amount of line runs out to absorb the shock, then a spring-loaded brake stops the drum. Should the hook engage too heavy an object, the line runs out to a strand of smaller diameter which parts without ill effect on the aircraft.

Radar for the Medical Profession.—Detection of foreign bodies such as gallstones within human tissues, by means of a technique employing the radar principle, may be possible in the near future. The method, so far used only on animals, has been developed at the Naval Medical Research Institute in Bethesda, Md. It involves transmitting ultrasonic energy into body tissues and observing on a cathode-ray oscillograph screen the reflected and transformed waves from the foreign body. Distance of the echo from the initial pulse gives the depth of the foreign object in the tissue.

WORKHARDENING OF METALS—A GENERAL THEORY

BY

ALFRED M. FREUDENTHAL [1]

I. THE PLASTIC DEFORMATION OF METALS

When the yield limit of a metal specimen under load is exceeded, plastic flow under constant or fluctuating stress is more or less rapidly blocked by changes within the structure of the material. These changes are accompanied by a gradually increasing resistance to further plastic deformation, increasing indentation hardness and increasing fracture strength as well as by changes in density, electric conductivity, magnetic properties and resistance to wear. Whereas the elastic shear modulus remains practically unchanged, slight changes may occur in the value of the bulk modulus and Poisson's ratio (1).[2] This process of changing the mechanical properties of metals by plastic deformation is usually termed strain-hardening or workhardening; the latter term is the more adequate since the changes may be related to an input of work. A general theory of the phenomenon can be developed on the basis of this relation.

The rate of change of the mechanical and other properties is related to the rate of irrecoverable change in the internal structure of the metal; it depends upon the undeformed structure of the material and is different for single crystals and for polycrystalline aggregates of various initial structure, that is, of structures of various grain size. It also depends upon the temperature and the speed at which the material is deformed.

Thus changes in the physical properties by workhardening are related to some modification that has occurred within the configuration of the crystal grains. Since mechanical properties of polycrystalline metals also depend very strongly upon the nature of the grain boundaries, changes of those properties are necessarily related to changes in the character and extent of these boundaries. The specific character of the grain boundaries is due to the distorted atomic arrangement produced by the competing claims as to the lattice to be formed exerted by the adjacent crystal grains on the small number of atoms located within the boundary region between the grains. The arrangement of those atoms is therefore as nearly "disordered" as is the arrangement of particles in an amorphous material, with the result that grain boundaries, as well as distorted atomic layers in general, behave in a way similar to amorphous materials; they show a highly temperature- and time-sensitive "viscous" response to loads.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

When a metal crystal is deformed, slip takes place along the eligible atomic planes which are most favorably oriented with respect to the direction of principal shear of the operative stress-system. The spacing of the slip planes and the extent of slip on individual planes have been found to depend upon the applied rate of strain or load (2). If the deformation is produced very slowly, the amount of slip on individual planes is extremely small and the number of planes correspondingly large; under such conditions individual slip is so uniformly distributed and so small that it may become invisible under the microscope. Above a certain critical strain- or loading rate, on the other hand, the spacing of slip planes and the extent of slip along individual planes suddenly increase and slip bands formed of clusters of atomic slip planes become visible under the microscope. The critical strain or loading rate at which the finely distributed slip is transformed into one essentially concentrated within clusters of adjacent slip planes dividing the crystal into lamellas of finite thicknesses, depends upon the rate at which applied strain energy is dissipated within the intercrystalline regions. If the rate of application of strain energy is higher than its rate of dissipation, extensive breakdown of the cohesion will occur sooner or later within the intercrystalline boundaries, accompanied by an abrupt change in the distribution of the slip. It is reasonable to assume that the rather uniform restraint imposed upon the deformation of an individual crystal by slip (provided by the unbroken, viscously deforming grain boundaries) will favor a fine distribution of slip planes and small amounts of slip on individual planes; such distribution will enable the discontinuous deformation by slip to occur in the course of a relatively small viscous deformation (creep) of the restraining intercrystalline medium. The propagation of extensive slip on widely spaced clusters of glide planes on the other hand requires excessive and rapid deformation of the restraining medium leading to its local destruction. Thus, finely distributed slip is probably characteristic of slowly deforming, relatively fine-grained polycrystalline aggregates in which the crystals deform within the slowly yielding viscous boundaries; this cannot be expected to occur in single crystals, the deformation of which is unrestrained, and in which, therefore, slip is sudden and extensive; nor can it be expected in rapidly deformed or coarse-grained polycrystals, since the restraint imposed by the grain boundaries on the slip along widely spaced slip bands is rapidly removed by local destruction of the cohesion in the boundary regions. Glide planes are thus free to propagate through a number of neighboring crystal grains.

II. STRUCTURAL THEORIES OF WORKHARDENING

Slip is stopped by the distortion of the atomic structure around the advancing edge of the slip planes. The amount of distortion created during slip determines the rate of blocking along the slip planes. Over-

all plastic deformation resulting from short slip on many finely distributed slip planes will therefore be less rapidly blocked than plastic deformation resulting from extensive slip along widely spaced clusters of atomic slip planes (slip bands), which involves the creation and distortion in the course of the motion of glide lamellas of finite thickness between distorted slip bands.

However, the development of a blocking mechanism by distortion of the atomic arrangement within the slip bands may be responsible for workhardening on a macroscopic scale only within the range of relatively small plastic deformation. The change in resistance to deformation that can be explained by this mechanism alone is not of the same order of magnitude as the changes observed in most polycrystalline aggregates. The blocking mechanism developing on individual slip planes or slip bands may explain the workhardening observed in single crystals; however, such workhardening represents only part of the total workhardening effect within the polycrystalline aggregate. It is therefore not so much the blocking of slip within the slip planes but the blocking of this slip at the grain boundaries which is overcome by fragmentation, rotation and elastic distortion and bending of fragments in the course of the deformation which produces the observed workhardening effects of the polycrystal. The crystal fragments may be of different size and shape; it appears, however, that they cannot be broken down, at least not permanently, into smaller units than the limiting minimum size that is stable at the considered temperature of the deformation and which, according to almost all experiments performed on metals, is of an order of magnitude of 0.1 to $1\mu^3$ (3).

Crystal fragmentation is the principal structural change accompanying plastic deformation within the range of strains up to 15 or 20 per cent. Changes in the physical properties within this range are thus essentially due to the refinement of the initial crystal grains producing crystallites of more or less uniform limiting size, and to changes in the energy content of the intercrystalline boundaries.

The primary effect of grain refinement is to increase the specific distortional energy that can be reversibly stored up within an individual crystal grain, since, according to the v. Mises' yield condition, the yield limit is proportional to the sum of the squares of the principal shear strains. If the simplifying assumption is introduced that slip within an individual small grain of the aggregate is primarily the result of the energy which locally produces pure shear in the most eligible slip system of the considered crystal, the critical energy level at which slip is initiated along a single one of the family of eligible planes is represented by the shear energy which can be released by slip taking place over one atomic distance a which is the unit step or "quantum" of slip. Slip over one half only of that step requires energy application, the second half of the step towards the new equilibrium position occurs spontane-

ously after the activation energy between the two positions has been overcome. The critical energy level is therefore directly proportional to the square of the ratio $(a/2\lambda)$, where λ denotes the dimension of the crystal (Fig. 1).

The change in the energy content of the boundaries of the broken-up crystal structure is connected with a decrease in density of the order of magnitude of 0.1 to 1 per cent, accompanying the workhardening process of most polycrystalline metals; no measurable decrease in density can usually be found in cold-worked single crystals. In the course of the fragmentation of the crystal structure, a certain amount of disordered material, containing bonds of high energy content, is created within the glide-planes. This is the result of the distortion of the atomic structure within those planes by slip, rotation and bending of crystal fragments during deformation, as well as of the restrictions imposed, upon load release, on the recovery of shape of the elastically distorted crystal

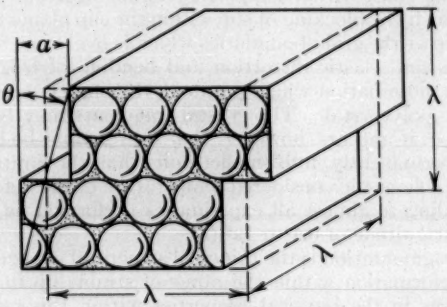

FIG. 1. Slip of a crystallite.

fragments by the preceding irrecoverable deformation of the polycrystalline aggregate.

To stabilize the fragmented and distorted crystalline structure a system of inter-atomic forces of very high potential energy is built up in the course of unloading. Thus a certain percentage of the volume of cold-worked, polycrystalline metals contains particles connected by bonds of very high energy content; the particles within this volume must be markedly further apart than those in the remaining crystalline volume. The potential energy introduced into and latently stored within the polycrystalline structure by the fragmentation of crystals is, therefore, probably contained within the small volume of crystal boundaries and of slip bands of distorted atomic structure.

This conclusion is borne out by the comparison of the changes in the lattice parameter observed in the course of workhardening and the amount of potential energy latently stored up within the fragmented

structure. It has been found that the amounts of strain energy or of volume change necessary to produce an over-all elastic strain consistent with the observed change of the lattice parameter are much smaller than the actually measured latent energy or the observed density change. Therefore, latent energy and local density changes must be concentrated mainly within the intercrystalline boundaries and within the distorted glide planes (4).

When an advanced stage is reached in the process of crystal fragmentation without fracture having occurred, the mechanism of plastic deformation by slip and fragmentation changes and gradually becomes one of formation of texture (orientation) by rotation and by temporary break-up of fragments which are subsequently reformed in the direction of the largest strain velocity. Changes in mechanical and other properties, produced during this stage of the deformation, are an expression of the developing anisotropy of the material; the structural pattern defined by grain size and by the extent of grain boundaries remains practically unaffected, and the hardness increase during this stage is therefore relatively small.

Four different mechanisms may thus be responsible for the increase of resistance to plastic deformation of a polycrystalline metal aggregate stressed above the yield limit. Although it has frequently been attempted to explain the workhardening phenomenon in terms of any one of those mechanisms alone, it is probably the joint or the successive operation of all four mechanisms which is responsible for the observed workhardening effects over the whole range of deformations of metals.

III. A GENERAL LAW OF WORKHARDENING

On the basis of the preceding discussion, workhardening is the result of:

(*a*) The increase in resistance to slip within a single crystal, due to the creation and propagation, under applied shear, of distortions of the atomic lattice, within the slip planes, called "dislocations." (Taylor's Theory.)

(*b*) The increase in resistance to plastic deformation of a polycrystalline aggregate produced by the fragmentation, along slip planes, of crystals and the rotation, elastic distortion and bending of the crystal fragments. The increased resistance is due primarily to the fact that the specific shear energy required to initiate slip in a single crystal is an inverse function of the square of the crystal size. (Bragg's Theory.)

(*c*) The stabilization of the fragmented and distorted crystal structure by a system of micro-residual (textural) stresses set up during unloading after fragmentation. This stress system stabilizes the distorted crystalline structure of the aggregate by introducing into the material a certain amount of "latent" potential energy, which becomes a characteristic feature of the workhardened polycrystalline pattern.

(*d*) Anisotropic change in resistance to plastic deformation produced by practically volume-constant deformation associated with rotation, break-up and immediate reformation of crystal fragments of limiting size in the direction of the maximum strain velocity.

Although these four workhardening mechanisms are assumed to be jointly operative in polycrystalline materials, the degree of their relative importance probably changes with the initial properties of the material, and with the progress of the workhardening process.

The propagation of dislocations [Mechanism (*a*)] is probably responsible for the initial workhardening associated with small plastic deformation, or for the workhardening of polycrystals consisting of a very small number of crystals. Beyond the range of small deformation the workhardening effects resulting from crystal fragmentation and storing up of "latent" energy [Mechanism (*b*) and (*c*)] may be assumed to be mainly responsible for producing the observable relations of stress and strain. Reorientation of fragments which does not produce appreciable hardening is operative in the formation of texture under large uni-directional strains; therefore, in polycrystalline metals the texture-forming mechanism comes into play at a late stage, when crystal fragmentation has been nearly completed, and further deformation can only proceed by directional reorientation of the crystal structure under conditions of considerable thermal instability.

Without attempting to discuss the relative merits of the mechanisms of dislocations and of fragmentation, which are not considered here as alternative but as jointly operative mechanisms, a feature common to both mechanisms will be used to develop a general function of workhardening. This feature is the dependence of the resistance to slip initiation within a crystal or within a polycrystalline aggregate, on an inverse function of a geometrical parameter λ, which is either the range over which a dislocation can migrate (its so-called "mean free path") or the range over which slip can take place in a crystal. The parameter λ can therefore be considered as a measure of the order of magnitude of the average linear dimension of the perfect crystals, or of the crystallites, forming the aggregate.

Considering an elementary crystal cube of dimension λ built up of atoms (Fig. 1) under conditions of pure shear, slip starts by the gliding of one atomic plane over another, with accompanying release of potential energy, when the elastic shear strain g has attained the value $a/2\lambda$; this is equivalent to an elastic displacement of the upper plane against the base plane of one half the atomic distance a. Hence, the elastic energy per unit volume which can be reversibly stored up before slip starts, the "resilience" $\frac{1}{2}Gg^2_{\searrow} = \text{const.}\,(1/\lambda)^2$, is inversely proportional to the square of the parameter λ. It is this limit of elastic energy that provides a measure of the "hardness" H by delimiting the energy level at which plastic deformation starts.

On the other hand, the theory of dislocations as developed by G. I. Taylor (5) leads to the "parabolic" relation of stress and strain.

$$s = \text{const.} \sqrt{g/\lambda} \tag{1}$$

according to which the "critical stress" at constant strain g at which slip proceeds, is inversely proportional to $\sqrt{\lambda}$. In terms of energy the "resilience" H, which is proportional to the square of stress, would be inversely proportional to λ.

If it is assumed that the increase in hardness, defined as increase in the resistance to plastic deformation in terms of energy, is not or not only, due to the "resilience" of individual crystals increasing with decreasing parameter λ, but to the storing-up of "latent" energy within the intercrystalline boundaries and distorted atomic layers of the fragmented polycrystal, the hardness increase dH would be proportional to the increase, in the course of fragmentation, of intercrystalline surfaces within the unit volume of the aggregate. If m crystals of parameter λ_m, making up the unit volume of hardness H_m are broken up into a unit volume of hardness H_n consisting of n crystallites of parameter λ_n, and if the ratio of hardness before and after fragmentation H_m/H_n is assumed to be proportional to the ratio of intercrystalline surfaces A_m and A_n per unit volume before and after fragmentation respectively, this hardness ratio may be expressed by

$$H_n/H_m = \frac{n \cdot \lambda_n^2}{m \cdot \lambda_m^2} = \frac{\lambda_m}{\lambda_n}, \tag{2}$$

since $m \cdot \lambda_m^3 = n \cdot \lambda_n^3 =$ unit volume. The hardness H in terms of energy is therefore inversely proportional to the parameter λ. This is the same relation that has been obtained from the dislocation theory.

Thus, according to any one of the three workhardening mechanisms, the hardness defined in terms of the strain energy required to initiate plastic deformation is inversely proportional to either the square of the parameter λ or to λ itself or to some intermediate power λ^n, where $1 < n < 2$. It is on the basis of this conclusion that a general law of workhardening can be developed. This law describes the change of state in terms of the process of energy transformation accompanying the changes in the structural pattern of the metal. It can therefore be expressed in terms of the basic thermodynamic equations

$$\frac{dW_s}{dt} = \frac{d\Phi_0}{dt} + \frac{dW_D}{dt}, \tag{3}$$

and

$$\frac{dW_D}{dt} = \frac{1}{T}\frac{dS}{dt} \geqq 0, \tag{4}$$

where W_s, Φ_0, W_D, S, and T denote the applied strain energy, the reversibly stored distortional energy, the dissipated energy, the entropy

and the absolute temperature, respectively. The > and = signs in Eq. 4 define irreversible and reversible processes, respectively. For every structural pattern defined by the energy W_D irrecoverably expended in producing it, a limiting amount of distortional energy $\Phi_0 = H$ exists up to which $dW_D/dt = 0$. The function $\Phi_0 = H = f(W_D)$ represents the basic workhardening relation.

Since energy is an additive quantity, the hardness of the polycrystalline aggregate can be assumed to be roughly the sum of the individual resilience values of the constituent crystals; hence

$$\tfrac{1}{2} \sum^{n} G \cdot g_m^2 \cdot V_m = \Phi_0(g) \cdot V = \tfrac{1}{2} G \cdot g^2 \cdot V, \tag{5}$$

where g denotes the limiting shear-strain and V the volume of the aggregate, while V_m and g_m respectively denote the volume and limiting shear-strain of an individual crystal of size m out of the n sizes of crystals forming the aggregate.

Since the hardness H of the aggregate is thus made up of the sum of the hardnesses H_m of the constituent crystal sizes, the contribution of the crystal size m to the hardness of the aggregate will necessarily be $H_m(V_m/V]$. If the structural pattern producing the hardness H is defined by a certain volume distribution of crystal parameters λ running from λ_l to λ_n, where λ_l denotes the largest size, λ_n the smallest (limiting) size, this pattern is related to the energy W_D irrecoverably expended in producing it from the initial structural pattern of the aggregate, defined by the initial hardness H_0.

The validity of this relation is however limited to conditions for which crystal fragmentation and distortion are the principal operating dissipation mechanisms; this is the case at temperatures at which the thermal stability of the distorted crystal pattern is high and effects of anisotropic reorientation are slight. At temperatures below recrystallization temperature, the energy dissipated by alternative mechanisms will, in general, be negligible. At temperatures near recrystallization temperatures, however, the amount of energy dissipated directly into heat, without producing an identifiable permanent change of structural pattern, that would result in increased hardness, becomes so large that the one-valued relation between hardness H and dissipated energy W_D no longer exists. Similarly, energy is dissipated without producing appreciable change in hardenss, if during deformation directional reorientation is the predominant effect. However, conditions of marked thermal instability of the fragmented structure at room temperature can be expected but at the approach to the upper limit of hardness. Since for most of the structural metals at room temperatures (or even at moderately elevated temperatures) fracture will intervene before the upper limit of workhardening is attained, the one-valued relation between hardness H and dissipated strain energy W_D may be assumed to

hold with fair approximation within the practically important range of deformation.

An increase in hardness dH is brought about by expending the energy dW_D to change to volume distribution of crystal sizes through the fragmentation of a part v_m of the total volume V_m of crystals of size λ_m and hardness H_m, into an equal volume of crystals of size λ_n and hardness H_n. Therefore the rate of workhardening of the aggregate

$$\frac{dH}{dW_D} = \sum_1^m (H_n - H_m) \frac{dv_m}{dW_D} \tag{6}$$

if the considered volume of the aggregate V is unity; the sum is to be taken over all crystal sizes affected, that is over all sizes $m > n$.

Fragmentation of crystals may be either gradual, through consecutive stages, when every crystal size is broken up into a smaller, but not necessarily limiting size, or "catastrophic," when every crystal size is shattered directly into fragments of limiting size, or it may be of an intermediate nature. Since it has been found that the first assumption leads to workhardening functions inconsistent with observation (6), the catastrophic type of fragmentation is assumed to be prevalent.

If q_m denotes the ratio of the volume of one crystallite of parameter λ_m to the volume of the crystal fragment of limiting parameter λ_n, the ratio of hardness of the two crystal sizes

$$\frac{H_n}{H_m} = \left(\frac{\lambda_m}{\lambda_n}\right)^2 = q_{m,n}^{\frac{1}{3}} \tag{7}$$

if the hardness is defined by the "resilience" of crystal grains alone. If, on the other hand, it is assumed that the increase of hardness is not due to the increasing resilience of the refined crystal grains, but results from the "latent" energy stored up in the intercrystalline surfaces, the hardness ratio according to Eq. 2 is

$$\frac{H_n}{H_m} = \frac{\lambda_m}{\lambda_n} = q_{m,n}^{\frac{1}{3}}. \tag{8}$$

Adopting Eq. 7 and introducing it into Eq. 6 the rate of workhardening

$$\frac{dH}{dW_D} = \sum_1^m H_m \cdot (q_{m,n}^{\frac{1}{3}} - 1) \frac{dv_m}{dW_D}. \tag{9}$$

By integration,

$$H = \sum_1^m H_m \cdot (q_{m,n}^{\frac{1}{3}} - 1) \cdot v_m + \text{const.} \tag{10}$$

If v_m denotes the relative volume of grains of parameter λ_m fragmented to date out of the volume V_{0m} of such crystals initially existing in the unit volume V, the rate of fragmentation of grains of any particular

parameter λ_m may reasonably be assumed proportional to the relative volume of such grains $V_m = (V_{m0} - v_m)$, existing at the considered stage of fragmentation defined by W_D. This assumption is generally used in all physical disintegration (decay) processes. Hence

$$\frac{dv_m}{dW_D} = (V_{m0} - v_m)\frac{1}{\alpha_m}, \tag{11}$$

where α_m is a factor of proportionality characteristic for the stability of the grain size; its inverse value is proportional to the rate of disintegration. With the condition that $v_m = 0$ for $W_D = W_{D0m}$, integration of Eq. 11 gives

$$v_m = V_{m0}[1 - e^{-(W_D - W_{D0m})/\alpha_m}], \tag{12}$$

valid for $W_D > W_{D0m}$.

The workhardening curve corresponding to Eq. 10 thus has different ranges governed by different equations, and delimited by the values of W_{D0m}. In the first range, from $W_{D01} = 0$ to W_{D02} the largest size of crystals of parameter λ_1 and hardness H_1 is fragmented into crystallites of limiting size, until the over-all hardness H of the polycrystalline aggregate (which, in the initial stage, that is, for $W_D = 0$ is $H = H_0 = H_1$) reaches the hardness H_2 of the grains of parameter λ_2. From there on the fragmentation of the crystal size of parameter λ_2 sets in, proceeding simultaneously with the continued breaking up of still existing grains of parameter λ_1 and so on. Thus for the first stage ($0 < W_D < W_{D02}$), with $H = H_1$ for $W_D = 0$:

$$H = H_1[1 + (q_{1,n}^{\frac{1}{3}} - 1)\cdot V_{10}(1 - e^{-W_D/\alpha_1})]. \tag{13}$$

For the second stage ($W_{D02} < W_D < W_{D03}$) with $H = H_2$ for $W_D = W_{D02}$:

$$H = H_1\{q_{1,2}^{\frac{1}{3}}[1 + (q_{2,n}^{\frac{1}{3}} - 1)\cdot V_{20}(1 - e^{-(W_D - W_{D02})/\alpha_2}] + (q_{1,n}^{\frac{1}{3}} - 1)\cdot V_{10}(e^{-W_{D02}/\alpha_1} - e^{-W_D/\alpha_1})\}, \tag{14}$$

where $q_{1,2}^{\frac{1}{3}} = H_2/H_1$.

Generally, the hardness $H(W_{Dm})$ within the m-th stage of fragmentation

$$H(W_{Dm}) = H_1\cdot\{q_{1,m}^{\frac{1}{3}} + \sum_{k=1}^{k=m}[q_{1,k}^{\frac{1}{3}}(q_{k,n}^{\frac{1}{3}} - 1)V_{k0}\cdot(e^{-(W_{D0m} - W_{D0k})/\alpha_k} - e^{-(W_{Dm} - W_{D0k})/\alpha_k})]\}, \tag{15}$$

where $q_{1,m}^{\frac{1}{3}} = H_m/H_1$ and $q_{k,n}^{\frac{1}{3}} = H_n/H_k$, H_n being the hardness of crystallites of limiting size λ_n. The rate of workhardening within this range

$$\frac{dH(W_{Dm})}{dW_{Dm}} = H_1\sum_{k=1}^{k=m}\left[q_{1,k}^{\frac{1}{3}}\cdot(q_{k,n}^{\frac{1}{3}} - 1)\cdot\frac{V_{k0}}{\alpha_k}\cdot e^{-(W_D - W_{D0k})/\alpha_k}\right]. \tag{16}$$

If the rate of workhardening is plotted against hardness, a straight line

$$\frac{dH}{dW_D} = C_1 + C_2 H \tag{17}$$

is obtained if one term only ($k = 1$) exists in Eq. 15 or, alternately, if α_k is the same for all crystal sizes. The shape of the function $H = f(dH/dW_D)$ thus provides an indication of the character of the process of fragmentation.

It is evident that the only change introduced into Eqs. 13 to 17 by adopting a different assumption concerning the operative workhardening mechanism would be to change the powers of all $q_{k.m}$- and $q_{k.n}$-ratios in those equations from $\frac{2}{3}$ to $\frac{1}{3}$, as required by Eq. 8 which would replace Eq. 7, on the basis of which the workhardening law (15) has been derived. This change is a change in constants only and does not affect the form of the general workhardening function.

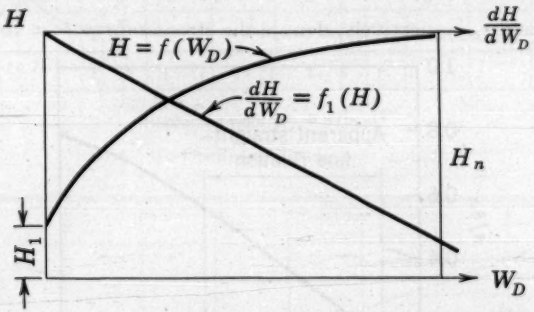

FIG. 2. Workhardening function for fragmentation of single crystal size.

IV. INTERPRETATION OF EXPERIMENTS

In applying Eq. 15 to the interpretation of experimental results under general conditions of stress, the proper measure of H and of W_D must be introduced. The elastic distortional energy is proportional to the second invariant of the deviator of stress I_{02s} or of elastic strain I_{02e}. There is, however, no simple measure of W_D in the case of a general state of stress. In the case of homogeneous stress, W_D can be directly computed from the area under the stress-strain diagram. In first approximation, however, W_D for a general state of stress may be assumed to be proportional to the second invariant of the deviator of plastic strain. Thus, the general workhardening function $\Phi_0 = H = f(W_D)$ for a general state of stress has the invariant form:

$$I_{02e} = F(I_{02p}), \tag{18}$$

where I_{02e} and I_{02p} denote the second invariants of the deviator of elastic and of plastic strain respectively.

In the limiting case of an aggregate with a uniform grain size Eq. 13 (with $V_{10} = 1$) governs the entire range of deformation. Hence

$$H = H_n - (H_n - H_1) \cdot e^{-W_D/\alpha}, \tag{19}$$

and

$$\frac{dH}{dW_D} = \frac{1}{\alpha}(H_n - H). \tag{20}$$

Relations (19) and (20) are represented in Fig. 2. The slope $1/\alpha$ of the relation (20) is constant over the entire deformation range. This constancy is an indication that only one crystal size is being broken up.

Equation 19 can be written in terms of stresses if W_D is considered a function of the plastic strain $\ln p$ alone

$$\left(\frac{s}{s_\infty}\right)^2 = 1 - \left[1 - \left(\frac{s_1}{s_\infty}\right)^2\right] \cdot e^{-(\ln p)^2/\alpha}, \tag{21}$$

where s_∞ and s_1, respectively, denote the stresses for $p = \infty$ and $p = 0$.

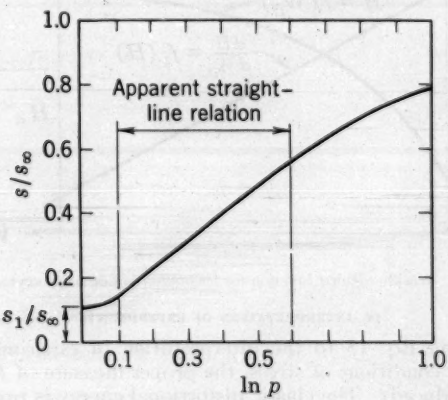

FIG. 3. Stress-strain diagram for workhardening process involving break-up of one crystal size (yield stress $S_1 = 0.1S$).

Hence

$$s = s_\infty \sqrt{1 - \left[1 - \left(\frac{s_1}{s_\infty}\right)^2\right] \cdot e^{-(\ln p)^2/\alpha}}. \tag{22}$$

The form of this relation for assumed values $\alpha = 1$ and $\frac{s_1}{s_\infty} = 0.1$ is represented in Fig. 3. The diagram shows that under the assumed conditions and within the range of the logarithmic plastic strains $0.1 < \ln p < 0.7$ the stress-strain diagram is very nearly a straight line;

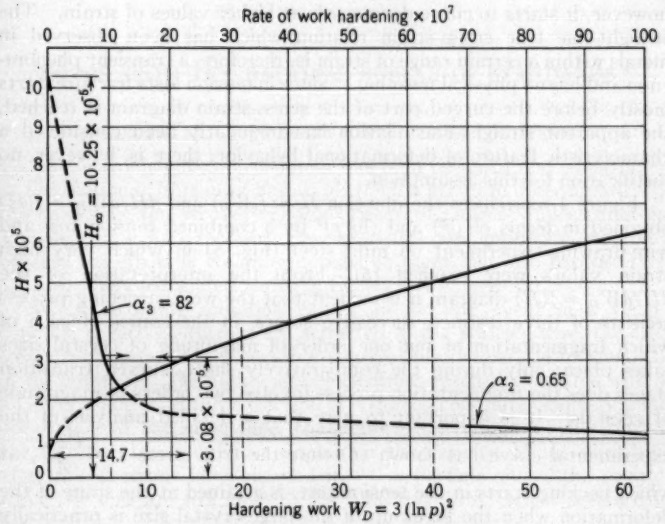

FIG. 4. Workhardening curve of mild steel (**8**).

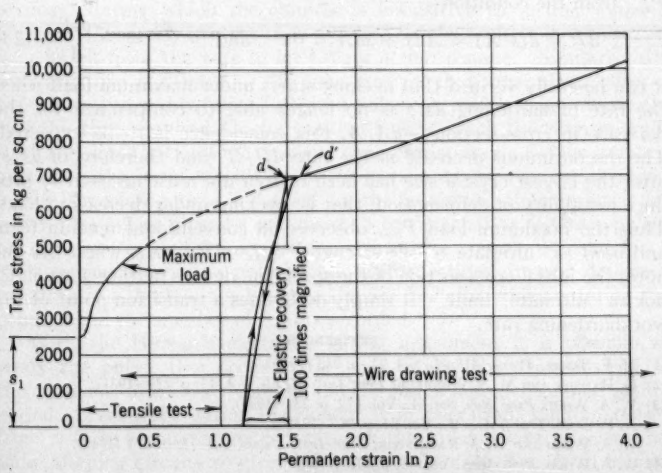

FIG. 5. Workhardening curve for mild steel established from tension test and successive drawing through dies (**8**).

however, it starts to curve downwards at higher values of strain. The straight-line true stress-strain relation which has been observed in metals within a certain range of strain is, therefore, a transient phenomenon and has no physical meaning. Since in tension tests fracture starts mostly before the curved part of the stress-strain diagram is reached, the apparent straight-line relation has frequently been considered a characteristic feature of deformational behavior; there is, however, no justification for this assumption.

Figure 4 reproduces the function $H = f(W_D)$ and $dH/dW_D = f(H)$ obtained in terms of (e^2) and $(\ln p)^2$ by a combined tension test and wire-drawing experiment on mild steel (Fig. 5) in which very high strain values were reached (6). From the interpretation of the $dH/dW_D = f(H)$ diagram it is evident that the workhardening process consists of three distinct, successive stages, in the course of each of which fragmentation of but one order of magnitude of crystal sizes takes place; only during the comparatively short, curved transition stages does the fragmentation process involve two orders of magnitude of crystals. It is important to note that a detailed analysis of the experimental curve has shown (6) that the true stress $s = \frac{P_{max}}{A}$, at which necking starts in the tension test, is attained at the stage of the deformation when the break-up of the large crystal size is practically completed. Since by computing the maximum load in the tension test P_{max} from the condition

$$dP = d(s \cdot A) = A\,ds + s\,dA = 0 \qquad \text{and} \qquad \frac{ds}{s} = -\frac{dA}{A} \tag{23}$$

it can be easily verified that necking starts under maximum load when the rate of hardening ds/s is no longer able to compensate for the decrease in cross-section $-dA/A$, this coincidence is to be expected. The discontinuous decrease in the rate dH/H (and therefore of ds/s) after the largest crystal size has been broken up, must necessarily produce instability of deformation, that is, necking under decreasing load. Thus the maximum load P_{max} observed in conventional tension tests and used as "ultimate tensile strength" $s_{max} = P_{max}/A_0$, where A_0 denotes the initial cross-section of the specimen, defines neither a strength nor an "ultimate" limit. It simply designates a transition point of the workhardening rate.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

ELECTRONIC INSTRUMENT TESTS GEIGER COUNTERS

An electronic gating instrument has been designed and constructed at the National Bureau of Standards for the accurate determination of the deadtime and recovery characteristics of Geiger-Müller counters. The deadtime and recovery characteristics are important not only because the errors of counting depend on them, but also because they are intimately associated with the electrical discharge process.

When incoming radiation causes an ionizing event in a Geiger counter, negative electrons are attracted to the positive wire, the electron stream producing a negative voltage pulse across the terminals of of the counter. The highly mobile electrons reach the positive wire of a Geiger counter within a fraction of a microsecond after ionization has occurred. At the same time, the heavy positive ions moving toward the negative outer cylinder remain in the vicinity of the wire long after the electrons have reached it. If a particle enters the counter while the ions are still close to the collecting wire, Geiger action cannot occur.

The deadtime is defined as the time interval after a pulse has occurred during which the counter is insensitive to further ionizing events. Physically it is the time required for the positive ions to travel far enough from the wire to let Geiger action resume. Similarly, the recovery time is defined as the time interval that must elapse after a pulse has occurred before a full-sized pulse can again occur; physically it is the time required for the positive ions to reach the counter wall. Both the deadtime and the recovery characteristics of a Geiger counter, as well as the input sensitivity of the associated circuits, are involved in the "resolving time," defined as the minimum time interval by which two pulses must be separated to be detected as separate pulses by the counter and its accessories. Since the counter is dead for a short period after each pulse, some disintegrations are not detected, resulting in a discrepancy between the number of particles entering the counter and the number of counts recorded.

With the Bureau's electronic gating instrument it is possible to study the pulses that occur in the time interval between the end of the deadtime of a Geiger counter and the end of a predetermined gating period. When the gating instrument is in operation, random pulses from a Geiger counter are fed through the pulse height selector and pulse shaping circuits to give sharp negative pulses of uniform height.

* Communicated by the Director.

These negative pulses follow two paths: one through an amplifier to a heavily biased stage and the other through a delay circuit to a gating multivibrator. An initiating pulse entering the system finds Channel 1 blocked by the heavy bias, but by way of Channel 2 it trips the multivibrator and opens Channel 1 for pulses to pass. Because of the delay in Channel 2, Channel 1 is not opened until the initiating pulse has died away. The delay used is 15 microseconds. Subsequent pulses find Channel 1 open, go through it, and are counted. After a time determined by the circuit constants of the multivibrator, Channel 1 is blocked off again and the whole process is repeated. In effect, the system selects pairs of pulses with a time separation less than a selected value determined by the setting of the controls on the multivibrator, and one count is recorded for each such pair received.

EFFECT OF BORON ON THE STRUCTURE AND PROPERTIES OF CAST IRON

War-induced scarcities of elements such as chromium and manganese called attention to the possible use of boron to increase the hardness and wear resistance of cast iron. However, specific data regarding the effect of boron on the structure and properties of cast irons have been somewhat limited. At the National Bureau of Standards' experimental foundry, an extensive study was recently made of the effect of boron in plain gray cast irons. Useful information on solidification characteristics, strength, hardness, microstructure, and chilling properties was obtained. These data should be of value to those industries which produce cast iron for specialized uses requiring definite characteristics.

Cast iron consists essentially of graphite particles dispersed throughout a steel matrix. Ordinarily the fractured surface of the metal is gray, but when cast iron is cooled rapidly from the molten state, it chills, forming white iron. The core of such castings consists of soft gray iron while the outside portion is the more durable white iron. Because of its hardness and resistance to wear, white cast iron is used in such articles as street-car wheels and rolls for rolling mills. The brittleness of the iron chip and the lubricating properties of the graphite make gray iron easy to machine. The graphite particles also act as cushions against thermal shock and thus prevent crazing. For this reason, gray iron is the only material that has been successfully used on a large scale for brake shoes or brake drum linings, which must absorb tremendous energy, convert it to heat, and still not seize the wheel or axle.

In the Bureau's investigation, plain gray cast irons of nine different stocks were studied. The irons were melted in high-frequency induction furnaces, and boron in the form of ferro-boron was added to the molten metal. When examined under a microscope, the structure of irons with a relatively large boron content was found to consist essen-

tially of a matrix of cementite containing islands of pearlite as well as other structureless islands.

Transverse strength was determined by subjecting bars to a breaking test. A disk was cut from each broken transverse test bar and used for Brinell hardness determinations and for metallographic examination. Fractures could be classified as gray or white-network. For bars with gray fractures, an increase in boron content was accompanied by a small increase in the transverse strength, Brinell hardness, and relative modulus of elasticity of the sample. However, the appearance of a white network or white specks on the fractured surfaces of bars containing larger amounts of boron usually was accompanied by a drop in the transverse strength and deflection, and an increase in the Brinell hardness and relative modulus of elasticity.

An important property of cast iron is its ability to form white iron, when chilled. Depth of chill may be controlled by varying the cooling rate or by adjusting the composition of the iron. In the present study the whiteness of the fractured surface of plate and wedge specimens was taken as a criterion of the iron's susceptibility to chill. For all castings there was a pronounced increase in depth of chill with an increase in the boron content. This was found to be accompanied by a greater development of the white network in the gray portions of the castings.

In general, the results of the investigation show that boron increases the depth of chill of cast iron as well as its hardness. Boron was also found to decrease the size and quantity of graphite particles, to increase the free carbide content, and to produce an undercooling effect which becomes more pronounced with greater percentages of boron.

NBS LIQUID OXYGEN CONVERTER

A fully automatic liquid oxygen converter having significant advantages for use in high altitude flight has been developed at the National Bureau of Standards. Its features include compactness, greatly reduced weight, sturdiness, simplicity of design and operation, rapid attainment of operating pressure, economy of oxygen, and delivery of warm gas.

The NBS Liquid Oxygen Converter consists of a standard 25 liter metal Dewar flask modified by the addition of a bottom drain, together with two coils, one for build-up of pressure and one for warming the gas as it is delivered. Four valves, two on the flask, one in the build-up tube, and the other in the delivery line, prevent excessive rise in pressure.

When oxygen gas pressure is to be built up, liquid is directed through the bottom drain and a liquid trap into the build-up coil, in which it is evaporated and warmed by the atmosphere. The warm gas rises through the coil by thermal convection and is then returned to the top

of the flask, where it mixes with the gas already present above the liquid. Gas pressure rises quickly as this cycle continues. After pressure has been established, oxygen may be withdrawn by forcing liquid through the bottom drain into the withdrawal coil, where it is evaporated and warmed by the atmosphere as it passes to the outlet mechanism.

The total converter unit is 25 in. high and 18 in. in diameter. It weighs 60 lb. and holds 62 lb. of oxygen, enough for ten men for 10 hr. To supply this quantity of oxygen from gas cylinders at 1800 psi. would require 21 tanks of 514 cu. in. each, which weigh a total of 350 lb. when empty. For flows up to 150 liters per minute (5 cu. ft. per minute), gas is delivered within 5° C. of atmospheric temperatures. Greater flow rates may be obtained, but the delivered gas is colder. Pressure may be built up, when the container is full, from 0 to 65 psi. in 10 sec.

The converter has successfully withstood tests under conditions of severe vibration (3000 cycles per minute), rapid acceleration (nine times the acceleration of gravity), extreme cold (−40° C.), and severe heat (75° C.). Performance at tilts up to 45 deg. was not materially different from that observed in the erect position. Converters built on the NBS design by commercial manufacturers have operated satisfactorily in flight tests at 40,000 ft.

In reliability and safety the new converter compares favorably with high-pressure gaseous oxygen systems. Wide variation is possible in container capacity, and in delivery flow, temperature, and pressure specifications. Thus, although designed specifically for aviation use, the device may be adapted with minor modifications to industrial and medical applications.

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ELECTRON-OPTICAL SHADOW METHOD

As the result of a series of electron-microscope experiments in the Electron Physics Laboratory of the National Bureau of Standards, an electron-optical shadow technique has been developed which provides a valuable tool for the quantitative study of electrostatic and magnetic fields of extremely small dimensions. The new development should also provide a powerful means for broadening present knowledge concerning space-charge fields, waveguide problems, and the basic magnetic properties of metals. Though similar in some respects to the electron-optical Schlieren method previously developed at the Bureau, the shadow method is much better adapted to precise determinations of field intensity.

In practice, the object to be studied, a recording wire for example, is placed between an electron source and a system of electron lenses. The lens system focuses the electron beam to form an image of the wire on a fluorescent screen. By placing a wire mesh of known gage just

beyond the back focus of the lens system, a shadow image of the mesh is superimposed on the image of the wire. The portions of the shadow network adjacent on the screen to magnetized regions of the recording wire are then found to show considerable distortion.

A complete theoretical analysis of this effect at the National Bureau of Standards has shown that the distortion of the shadow image is due to the deflection of the electron beam by the field of the recording wire at each magnetized region. The displacement and change in size of the shadow image depend on the strength of the field of the magnetized wire. Although it is possible to compute field intensity from the intensity distribution of the pattern obtained by the Schlieren method, the shadow method is of far greater utility for quantitative work since the image displacement and magnification can be measured much more accurately than can the intensity distribution across the Schlieren pattern on a photographic plate.

Perhaps the greatest value of the electron-optical shadow method lies in its utility for exploring complex electric and magnetic fields of extremely small dimensions or in which a probe of greater size than the electron would disturb the field under study. The shadow technique now provides data for accurate calculation of the absolute value of the intensity in the neighborhood of a specimen of any size or shape without altering or disturbing the field. The method is thus well adapted to investigation of the fundamental nature of ferromagnetism.

The Bureau is also applying the principle of the shadow technique to the study of spherical aberration in electron lenses. When a fine wire mesh is placed in the focal region of a lens having spherical aberration, the shadow image of the network is enlarged. The resultant pattern may be interpreted to give information of value in correcting the lens.

NBS METHOD FOR MICROSECTIONING

An improved and relatively rapid method for cutting extremely thin sections of biological tissue for study with either the light or electron microscope has been developed by the National Bureau of Standards. The application of the electron microscope to many biological problems has been seriously hampered by the lack of such a method, because of the very slight penetrating power of the electron beam in commercial instruments and the relatively great depth of field involved. Yet the techniques that have been available for preparing thin sections are quite elaborate and difficult, requiring expensive equipment and producing few usable sections.

An important feature of the Bureau's method is an easily made, inexpensive device which holds the embedded specimen and at the same time advances it gradually and uniformly toward the knife of a micro-

tome. The specimen is embedded in polybutyl methacrylate to produce an optically clear medium having highly desirable cutting properties. The material is fed to the microtome in extremely small increments by means of the thermal expansion of a metal specimen holder. The specimen holder is cooled by carbon dioxide gas and thus contracts. As the holder begins to warm and expand, it moves toward the cutting edge of the microtome. The length of time between cuts and the control of the carbon dioxide determine the thickness of the microsection. Thus, sections of tissue having undistorted structure and uniform thickness of fractions of a micron may be cut, one at a time.

Although the Bureau's new method for obtaining very thin sections has given excellent results, it possesses certain limitations. The greatest chance for failure appears to lie in the polymerization of the embedding mass. However, use of low-temperature catalysts will minimize difficulties at this stage. Efforts are now being made at the Bureau to apply the new sectioning method to the study of the structure of natural and artificial fibers found in leather, paper and textiles.

EFFECT OF SIMULATED SERVICE CONDITIONS ON PLASTICS

The use of laminated plastics in aircraft has increased during the past few years, especially in accessories and semi-structural parts, such as partitions, linings, propellers, and wing flaps. There has accordingly developed a need for more information regarding the action of the weather, temperature, and humidity in order to evaluate these plastics for aircraft applications and to prepare specifications.

To provide the necessary data, the National Bureau of Standards under the sponsorship of the National Advisory Committee for Aeronautics undertook an investigation to determine the effects of simulated service conditions on the weight, dimensions, and flexural properties of phenolic and unsaturated-polyester plastics. The test materials were commercial products and included nine laminated plastics which are the types commonly employed in aircraft.

For accelerated weather testing the specimens were subjected to cycles of ultraviolet light and fog while the accelerated service tests consisted of exposure to various temperatures and relative humidities. Three sets of specimens were exposed for one year to outdoor weathering. Changes in weight, dimensions, and flexural properties were the criteria used in analyzing the data gained. In most of the tests the changes in weight and dimensions were negative, any positive changes being in thickness. In several cases increases in flexural strength were noted and attributed to further cure of the resins.

THE FRANKLIN INSTITUTE

MEDAL DAY PROCEEDINGS

The annual reception, dinner, and presentation of awards, known as "Medal Day," took place at The Franklin Institute on Wednesday, October 19, 1949. The reception began at 6:30 P.M. and dinner was served at 7:20 P.M. Mr. Richard T. Nalle, President of the Institute, presided at the dinner and presented the subsequent awards.

Reception

The reception preceding the dinner was given by the Hostess Committee of The Franklin Institute, and was attended by 294 persons. Immediately following the reception and just before dinner was served in Franklin Hall, Mr. Nalle requested the guests to face the flag above the Franklin statue while Mr. Guy Marriner, director of music at The Institute, played the National Anthem.

Remarks of the President

Mr. Nalle: "Tonight we honor a group of individuals who, by dedicating their talents and energies to the investigation of scientific problems, have furthered the value of science in our daily lives.

"The curiosity which prompts such investigation, and the practical channelling of that curiosity, is assuredly in line with Benjamin Franklin's character. That early American gentleman had the most profound respect for every phase of science, but most especially for its practical application to living. He would certainly approve of the work of the scientists gathered here.

"Therefore, may I propose, ladies and gentlemen, a toast on this Medal Day to the living memory of Benjamin Franklin."

* * *

Mr. Nalle (*continued*): "On behalf of the Board of Managers and Officers of The Franklin Institute, I welcome all of you here tonight where we have gathered to honor thirteen men for their achievements in the world of Science and the Arts.

"Our special thanks are due the Hostess Committee. They have been responsible for the flowers and have given generous assistance to the reception."

Stated Monthly Meeting

Mr. Nalle (*continued*): "Tonight is also the regular stated October meeting of the Institute. The minutes of the last stated meeting in May have been published in the June issue of the Journal of The

FRANKLIN INSTITUTE and, if there are no corrections or additions proposed at this time, they are declared approved as printed.

"It is customary to elect recipients of the Franklin Medal to Honorary Membership in The Franklin Institute. Therefore we would like to hear a motion according this privilege to Dr. The Svedberg, of Upsala, Sweden, who is the 1949 Franklin Medalist."

A motion was made by Dr. Allen and duly seconded.

MR. NALLE (*continued*): "It has been moved and seconded that Dr. Svedberg be elected an honorary member of The Franklin Institute. Will all members of The Franklin Institute present, who are in favor signify by saying 'Aye'? Contrary minded—.

"The motion is carried and I declare Dr. Svedberg elected to Honorary Membership."

* * *

MR. NALLE (*continued*): "Among us tonight are twenty-three former recipients of Medals given by The Franklin Institute. As I mention the names, cities and Awards, will each of you please rise and remain standing until the full roll of former Medalists is completed? To save time, I suggest that our applause be withheld until the names of all these gentlemen have been called.

"These Medalists are:

FORMER MEDALISTS ATTENDING MEDAL DAY—1949.

Year	Medalist	City	Medal
1921	Adams, Dr. L. H.	Washington, D. C.	Longstreth
1905	Alteneder, Mr. Theodore G.	Philadelphia, Pa.	Longstreth
1948	Arone, Mr. Nicholas F.	Philadelphia, Pa.	Longstreth
1930	Bailey, Mr. Ervin G.	New York, N. Y.	Longstreth
1947	Berman, Mr. Samuel	New York, N. Y.	Longstreth
1947	Billner, Mr. Karl P.	Philadelphia, Pa.	Brown
1904	Clamer, Dr. G. H.	Philadelphia, Pa.	Cresson
1947	Davidson, Mr. Kenneth S. M.	Hoboken, N. J.	Wetherill
1946	Dodge, Mr. Ronald D.	Poughkeepsie, N. Y.	Certificate of Merit
1937	Eksergian, Dr. Rupen	Philadelphia, Pa.	Henderson
1945	Eksergian, Dr. Rupen	Philadelphia, Pa.	Levy
1936	Hall, Mr. Peter P. G.	Philadelphia, Pa.	Longstreth
1943	Heyl, Mr. Paul Renno	Washington, D. C.	Potts
1947	Kent, Mr. Robert H.	Aberdeen, Maryland	Potts
1948	Lovell, Dr. Clarence A.	Summit, N. J.	Potts
1924	McBride, Mr. Thomas C.	Philadelphia, Pa.	Longstreth
1925	Nachod, Mr. Carl P.	Birmingham, N. J.	Longstreth
1923	Parks, Mr. Harry S.	Pottstown, Pa.	Longstreth
1948	Rajchman, Dr. Jan A.	Princeton, N. J.	Levy
1946	Southworth, Dr. George	New York, N. Y.	Levy
1947	Southworth, Dr. George	New York, N. Y.	Ballantine
1932	Wenner, Dr. Frank	Washington, D. C.	Wetherill
1939	Wilford, Mr. E. Burke	Philadelphia, Pa.	Certificate of Merit
1941	Wilson, Dr. Benjamin J.	Oreland, Pa.	Longstreth
1915	Young, Brig. Gen. Charles D.	Chester County, Pa.	Longstreth
1948	Young, Brig. Gen. Charles D.	Chester County, Pa.	Henderson

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MR. NALLE (*continued*): "Thank you, gentlemen. Now, the next Order of Business is a report from the Executive Vice President and Secretary of the Institute, Dr. Henry Butler Allen. Dr. Allen, will you please make your report?"

REPORT OF THE SECRETARY

"The Institute in 1949"

DR. ALLEN: "Mr. President, Honored Guests, Members and Friends of the Institute. The year 1949 has seen the passing of three beloved members of our Board of Managers, who for many years have been with us on Medal Day: Mr. Malcolm Lloyd, Jr.; Mr. James S. Rogers; and Mr. Philip C. Staples, past president of the Institute.

"You will have seen at your tables this evening a simple little folder titled 'This Is The Franklin Institute.' Most of you know that the Institute's activities embrace the Benjamin Franklin Memorial, the JOURNAL OF THE FRANKLIN INSTITUTE, Lecture and Meeting series, a technical Library, the Science Museum and Planetarium, the Bartol Foundation, the Industrial Research Laboratories; and tonight you are clearly made aware of our Medal Award activity.

"But even though we are this year marking our 125th anniversary, there are quite a few other people around who still are not aware of this rather wide range of activities, most of which have existed since we were founded in 1824, 'For the Promotion of the Mechanic Arts'—as the physical sciences and engineering were then known. Hence this little folder which is being widely distributed.

"This year has seen some growth despite uncomfortably restricted means. We have had some notable gifts, such as Benjamin Franklin's 1757 holograph Will, given us by Mary Curtis Zimbalist in memory of her father Cyrus H. K. Curtis, a past benefactor. Through the aid of industry we have markedly improved the electrical section in our Museum. And a real start is being made toward a better exhibit of the science behind one branch of electrical communications through an outstanding exhibit of telephony now being built for us by the Bell Telephone Company. An intriguing illusions exhibit has been installed where you can go into a room in which your normal ideas about gravity seem to be proven all wrong. A visitors' lounge now exists where one can give his or her museum feet a rest while he or she smokes and sees a televised football game.

"New knowledge has been gained about cosmic rays at our Bartol Laboratories where a world's altitude record has been made by one of their cosmic ray captive balloons. Through their separate trust funds there is under construction a building near Swarthmore to house a new large Van de Graaff machine for studying the atom.

"The industrial Laboratories for Research and Development in which 230 people are employed have been getting useful results, in-

cluding, for example, aid for industry through a sonic testing device for grading grinding wheels; and a reading device for aiding the many thousands of semi-blind people, under a grant from the Kellogg Foundation.

"Our Library, besides keeping up to date through current technical books and periodicals has put on display a noteworthy collection of Franklin imprints given us by Mr. Samuel H. McVitty.

"Our Lecture series was highlighted by the first talk in the newly created Edward G. Budd Lectureship, endowed by his family in memory of that outstanding Philadelphia manufacturer, a former member of our Board, and one who always claimed that he got his technical start in our old Franklin Institute schools. And our 125th Anniversary year, together with the 200th Anniversary of Franklin's classic electrical experiments (which today would be called 'research') were celebrated by a day and evening special meeting. At that time we devoted the afternoon to the Franklin Anniversary. A talk was given by Dr. Conklin, President of Franklin's own American Philosophical Society, another by our past Medalist Robert A. Millikan, on Franklin's work in electricity, and one by Dr. C. Guy Suits, Director of Research of the General Electric Company, on electrical research today. In the evening the Institute's anniversary was well recognized by inimitable remarks from our own Senator George Wharton Pepper, and through a delightful story of the Institute from its founding by a great grandson of its first Board Chairman, our own present fellow Board member, James H. Robins.

"And last but not least, we are back to peace times. Our Board is actively working to increase the numbers in our generous group now numbering about 225 people known as Friends of Franklin who give us life blood through current gifts of money, and to encourage larger gifts through bequests.

"Since last Medal Day three new members have joined our Board: Mr. Francis J. Chesterman, recently President of the Bell Telephone Company of Penna.; Mr. E. Paul du Pont, Engineer and Manufacturer of Montchanin, Del.; and Mr. S. W. Rolph, Executive Vice President of the Electric Storage Battery Company of Philadelphia.

"I have touched upon some of the highlights of our year 1949. We hope you can all visit us often. We want our Medalists to feel that this is their home when in Philadelphia. The Franklin Institute is more than just marble halls—it is, with its 5400 members all over the world, a living creative influence to thousands and thousands of young and old; worthy of your friendship and support."

MR. NALLE: "Thank you, Dr. Allen. And now, will the Sponsors for the 1949 Medal recipients please take their places?"

* * *

MR. NALLE (*continued*): "Most of our guests are aware that we celebrated our 125th year last spring on April 20th. Through that

century and a quarter, many amazing discoveries and inventions have been added to the world's accumulated knowledge. The Institute has rendered a public service by acting not only as custodian in our library and museum of the physical record and evidence of such scientific achievements, but by recognizing and rewarding outstanding individuals who have brought new things to the scientific world. Since 1835 we have been making awards upon the recommendations of our carefully chosen Committee on Science and the Arts.

"These awards have been established through the generosity of far-sighted men who wished to insure recognition over the years of the accomplishment of worthy scientific effort. The Science and Arts Committee carries the responsibility for choosing the recipients in accordance with the terms which accompany each award. Their time is devoted throughout the year to investigating outstanding developments in science in order to determine which cases specially merit recognition. Because of the care and integrity exercised by the Committee members for more than a hundred years, our awards have become internationally known and are highly regarded in all scientific fields.

"Each Medalist, when he comes to the Institute to receive his award, is sponsored by the Chairman of the Sub-Committee from the Science and Arts Committee which had recommended him. This Sponsor gives a brief report about the candidate's work ending with the citation.

"Tonight it is a matter of great regret that three of our Medalists are unable to be here. Dr. The Svedberg has just taken on the duties of Director of the newly created Gustaf Werner Institute for Nuclear Chemistry and therefore could not leave his country this month.

"Dr. Hume-Rothery also is absent because his duties at Oxford University prevent his leaving England just now.

"Mr. Millspaugh, upon arrival in Philadelphia, was, unfortunately, taken ill and is in the Pennsylvania Hospital.

"An invitation was extended to His Excellency, Erik Boheman, Swedish Ambassador to the United States, to receive the Franklin Medal and accompanying Report and Certificates for Dr. Svedberg. Greatly regretting his inability to be present, he delegated Mr. Ingemar Hägglöf, Counselor of the Embassy, to represent him.

"Sir Oliver Franks, British Ambassador to the United States, having likewise been invited to receive the Hume-Rothery award, being unable, unfortunately, to be present, duly designated Dr. William Angus Macfarlane, Scientific Attaché of the Embassy, to act in his behalf.

"The awards to the first two Medalists will be duly transmitted to them after presentation here tonight. That to Mr. Millspaugh will be taken to him as soon as circumstances permit."

PRESENTATION OF THE LONGSTRETH MEDALS

(To Arthur M. Young, Counsulting Engineer, New York City, N. Y.; William H. Millspaugh, President and Director, Centrifugal Steel, Inc., Sandusky, Ohio.)

Arthur M. Young

William H. Millspaugh

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MR. NALLE: "I recognize Mr. Ralph H. McClarren, the Sponsor of Mr. Arthur M. Young."

MR. MCCLARREN: "Mr. President, I present Arthur M. Young for an award.

"Since 1927, he has devoted most of his time to the solution of problems pertaining to vertical flight. The well known Bell Helicopter is one major result of his work. Incorporated in the Bell Helicopter is a special mechanism which provides stability in flight.

"Like the airplane, the helicopter will be unstable in flight unless specific means are provided for stabilizing it. The ability of a pilot to keep his helicopter on an even keel, especially when hovering in gusty air, depends upon the degree of stability provided.

"Mr. Young has made a contribution toward improving this stability and has applied it primarily to the single rotor helicopter.

"In developing and improving his ideas, he made use of small scale flying models. This led him into designing, building and flying remote control models, the application of which proved unique and most valuable in advancing his theoretical and practical work.

"I present Arthur M. Young, of Paoli, Pennsylvania, as a candidate for an Edward Longstreth Medal, 'In recognition of his successful application of flying scale models to test unproven ideas in helicopter flight.'"

MR. NALLE: "Mr. Young, by virtue of the power vested in me as President of The Franklin Institute of the State of Pennsylvania, I present to you this Edward Longstreth Medal and the Certificate and Report which accompany it.

"I now recognize the next Sponsor, Dr. Benjamin J. Wilson,"

DR. WILSON: "Mr. President, I present, *in absentia*, William Hulse Millspaugh for an award.

"He has been largely responsible for improvements in paper-making machinery. By devising and successfully applying a method for removing water from the paper pulp by suction, instead of pressing between rolls as was done formerly, he secured, not only an increase in the rate of production, but an improvement in quality as well.

"He also pioneered in the practical application of the centrifugal casting of rolls, and succeeded in making rolls several times as long as was possible to secure by use of the old method of vertical sand-core casting. This permitted an increase in the paper width and accordingly, a further increase in the rate of production of several fold.

"I present, *in absentia*, William Hulse Millspaugh, of Sandusky, Ohio, as a candidate for an Edward Longstreth Medal, 'In consideration of his outstanding contributions to the art of paper making and to his introduction and promotion of the art of the centrifugal casting of rolls for Fourdrinier machines and other purposes.'"

MR. NALLE: "It is my privilege, by the same authority, to present also, to Mr. Millspaugh, *in absentia*, an Edward Longstreth Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE POTTS MEDALS

(To John William Mauchly, President, and J. Presper Eckert, Jr., Vice President, Eckert-Mauchly Computer Corporation, Philadelphia, Pa.; and Clinton Richards Hanna, Associate Director of Research, Westinghouse Electric Corporation, Pittsburgh, Pa.)

MR. NALLE: "I now call on Dr. Richard M. Sutton, Sponsor of Dr. Mauchly and Mr. Eckert."

DR. SUTTON: "Mr. President, I present John William Mauchly and J. Presper Eckert, Jr. for awards.

"These gentlemen are known for their distinguished work on the ENIAC, or the Electronic Numerical Integrator and Computer, a remarkable electronic calculating machine, conceived with great ingenuity and built on a bold scale at the University of Pennsylvania. Their work represents a significant combination of shrewd electronic design and sound engineering execution to create a computing machine of almost fabulous speed and accuracy.

"The ENIAC is now installed in the service of the Ordnance Department at the Aberdeen Proving Grounds where it daily performs

prodigious computing tasks. Capable of doing as many as 5000 additions of two ten-digit numbers in one second, its speed of handling numbers swiftly and accurately outstrips the imagination.

"I present John William Mauchly and J. Presper Eckert, Jr., both of Philadelphia, to each of whom has been awarded a Howard N. Potts Medal, 'In recognition of their design and construction of the ENIAC, the first large-scale, general purpose, digital electronic computer, a machine of great accuracy which makes feasible the application of mathematics to problems which hitherto had either to be ignored or else solved in a much more laborious way.'"

MR. NALLE: "By virtue of my power as president, I present to you, Dr. Mauchly, this Howard N. Potts Medal and the Certificate and Report which accompany it.

"Mr. Eckert, by the same authority, I hereby present to you also a Howard N. Potts Medal and the Certificate and Report which accompany it.

John William Mauchly

J. Presper Eckert, Jr.

"I recognize Dr. Benjamin J. Wilson, Sponsor of Mr. Hanna."

DR. WILSON: "Mr. President, I present Clinton Richards Hanna for an award.

"He has been largely responsible for inventing and developing a Tank Gun Stabilizer. Before the development of this device, it was customary in tank engagements for alternate tanks to halt in order to cover the advance of the rest of the column. While the halted tanks could aim and shoot their guns effectively, they became, when stationary, 'sitting ducks' for the enemy artillery and were easily put out of commission. It was necessary to use this method, however, because of the difficulty of aiming and shooting from a pitching and rolling tank in motion.

"The development of a gun stabilizer, however, changed this condition. This device keeps the gun aligned on the target despite the pitch and roll of the tank. In tests, the stabilizer enabled a gunner to obtain better than 70 per cent hits, over a variable range up to 1200 yards while traveling at approximately 15 miles per hour over cross-country terrain. Under similar conditions, without stabilizer, fewer than 1 per cent hits were possible, even with experienced gunners. A further advantage was that approximately three times as many sighted shots could be made in a given time, with stabilization.

"I present Clinton Richards Hanna, of Pittsburgh, Pennsylvania, as a candidate for a Howard N. Potts Medal, 'In recognition of his initia-

Clinton Richards Hanna

tive in the conception and development of the Tank Gun Stabilizer, the value of this device in attaining accuracy of fire while the tank is in motion over rough terrain and in securing a greater number of aimed shots than was formerly possible.'"

MR. NALLE: "By the same authority, I award you a Howard N. Potts Medal and Certificate and Report which accompany it."

PRESENTATION OF THE LEVY MEDAL

(To Alan Stewart FitzGerald, Consulting Engineer, San Francisco, Calif.)

MR. NALLE: "Dr. Boehne, the Chair recognizes you as the Sponsor for the Levy Medal recipient."

DR. BOEHNE: "Mr. President, I present Alan Stewart FitzGerald for an award.

"Magnetic amplifiers are suitable for a number of important applications for which electron tubes have hitherto been employed. Magnetic amplifiers, so applied, are superior to emission devices because of their greater reliability, longer life, and freedom from maintenance problems. For applications in which a speed of response only slightly less than instantaneous is adequate, magnetic amplifiers play an important role. These include the regulation of such quantities as temperature, voltage, speed, and are particularly useful tools in servo-mechanism techniques.

"The author presented and reviewed circuit arrangements and connections for combining a number of saturating reactors to form multistage magnetic amplifiers. This was followed by a procedure for computing the performance of a saturating reactor when used as a magnetic amplifier, so that the relation between input and output could be

Alan Stewart FitzGerald

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quantitatively expressed and indicated in the form of curves. The work outlines the principal elements in the design of magnetic amplifiers and concludes with a description of the design and construction of magnetic amplifiers of various types in which the performance characteristics are presented in curve and tabular form.

"I present Alan Stewart FitzGerald, of Los Angeles, California, as a candidate for the Louis Edward Levy Medal, 'In consideration of his papers on the subject of Magnetic Amplifiers, appearing in the October, November, and December 1947 issues of the JOURNAL OF THE FRANKLIN INSTITUTE.'"

MR. NALLE: "Mr. FitzGerald, by the same authority, I present to you the Louis E. Levy Medal, and the Certificate and Report which accompany it."

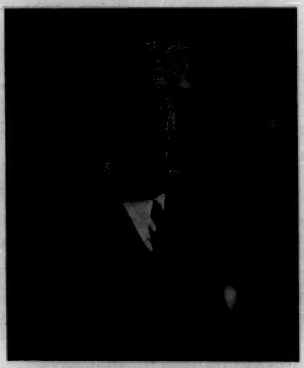
PRESENTATION OF THE HENDERSON MEDAL

(To John V. B. Duer (Retired), Pennsylvania Railroad, Fairfield, Conn.)

MR. NALLE: "I now call upon Dr. Rupen Eksergian."

DR. EKSERGIAN: "Mr. President, I present John Van Buren Duer for an award.

"The Pennsylvania Railroad electrification is the world's outstanding electrification, when measured by its power capacity and density of loading. The average power load is in the order of 150,000 KW. and the load factor around 70 per cent. Power is transmitted at 132,000 volts with a trolley voltage at 11,000 volts. The most powerful locomotives per unit weight are the famous G-G-1, developing a peak power at 8100 h.p. and a continuous rating at 4600 h.p.



John V. B. Duer

"While the electrification represents the culmination of a large background from previous electrifications, it is on the other hand unique in many electrical, mechanical and operational characteristics. It carries the densest railway traffic in the world.

"The over-all engineering and construction represents the efforts of a large group of supply companies and consulting engineers who contributed on electrical equipment, motive power and line construction.

"The final responsibility for the coordination of this work, however, rested with John Duer, who, as electrical engineer and later as Chief Electrical Engineer of the Pennsylvania Railroad, showed remarkable insight and engineering judgment in the coordination of the work and its ultimate accomplishment to actual operation.

"His later responsibilities as Senior Officer on over-all Railway Engineering matters covered research developments and studies on steam locomotive and car equipment, on traffic surveys, including special problems, such as the development of a steam turbine locomotive, a high speed freight car truck, the bonding of rails, and colloidal coal developments. His efforts on standardization, both in electrical and mechanical equipment are particularly noteworthy.

"I present John Van Buren Duer, of Fairfield, Connecticut, as a candidate for the George R. Henderson Medal, 'In consideration of his judgment and guidance in the coordination of engineering matters, including full technical responsibility in the electrification of the Pennsylvania Railroad, and in recognition of his contributions and resourcefulness in research developments in over-all railroad problems.'"

MR. NALLE: "Mr. Duer, by the same authority, I present to you the George R. Henderson Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE WETHERILL MEDALS

(To Harlan D. Fowler, Consulting Engineer, Whittier, Calif., Thomas L. Fawick, President, Fawick Airflex Company, Inc., Cleveland, Ohio; and Edgar Collins Bain, Vice President, Carnegie-Illinois Steel Corporation, Pittsburgh, Pa.)

MR. NALLE: "Mr. Fowler's Sponsor, Mr. LePage, who is also Chairman of the Committee on Science and the Arts, is now recognized."

MR. LEPAGE: "Mr. President, I present Harlan D. Fowler for an award.

"He has made an important contribution to the art of airplane design which has resulted in increasing the safety of flying and broadening the range of usefulness of the airplane, both in civilian service and as a weapon of warfare.

"The reconciliation of high top speed with safe and practical landing speed is, in aircraft design, one of the greatest single problems. Obviously, the minimum speed at which an airplane can sustain its weight is a function of its wing area. A plane with a large wing will, generally speaking, have a lower and, therefore, safer and more practical landing speed than a plane with a comparatively small wing. But such a machine will be handicapped in flight by too large and cumbersome a wing structure.

"The problems of structurally changing the wing area in flight are such as to render this obvious solution of the problem impractical.

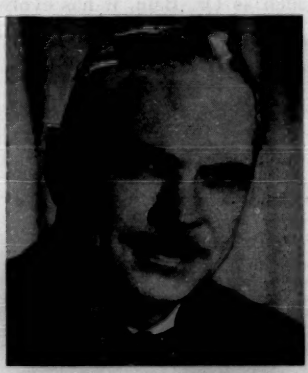
"Our Medalist has succeeded in developing a novel wing-flap device which, when incorporated in the wing structure of an airplane, constitutes the aerodynamic equivalent of variable wing area. His wing-flap principles have been adopted by several leading aircraft manufacturers

and those airplanes which incorporate these principles show remarkable speed range, from landing to top speed, and thereby possess performance characteristics which broaden their utility as well as their safety.

"I present Harlan D. Fowler, of Whittier, Calif., as a candidate for a John Price Wetherill Medal, 'In recognition of his many years of consistent endeavor directed toward a successful solution of the problem of variable lift airplane wings, culminating in the development of the Fowler Flap as an ingenious combination of variable wing area, variable wing camber and adjustable slot.'"

MR. NALLE: "By the same authority, I present to you a John Price Wetherill Medal, and the Certificate and Report which accompany it.

"Mr. Rogers, the Chair recognizes you as the Sponsor for Mr. Fawick."



Harlan D. Fowler

Thomas L. Fawick

MR. ROGERS: "Mr. President, I present Thomas L. Fawick for an award.

"The Fawick Airflex Clutch consists primarily of a pneumatic reinforced rubber bag, usually of flattened shape, mounted in an annular space between a driving and a driven unit, the bag being vulcanized to one of the units and brought in contact with the other unit by admitting air pressure to the bag during operation. The device may be used as a clutch, brake, or combination of both. As the medium for transmitting torque is flexible, shaft vibrations and operating shocks are reduced, slight misalignment of shafts corrected, uniform loading secured, and simplicity of design obtained by the use of a single moving element.

"I present Thomas L. Fawick, of Cleveland, Ohio, as a candidate for a John Price Wetherill Medal, 'In consideration of the adaptation

of the well-known qualities of rubber for expansion, flexibiliy and durability, to its use in the form of an inflatable tire or bag between two rotating members to act as a clutch, and its use between a rotating member and a stationary member to act as a brake, and the excellent design and construction of the details developed, resulting in a practical device, widely and successfully used in industry.'"

MR. NALLE: "Mr. Fawick, by the same authority, I present to you a John Price Wetherill Medal, and the Certificate and Report which accompany it.

"Mr. Francis Foley, will you please present your candidate?"

MR. FOLEY: "Mr. President, I present Edgar Collins Bain for an award.

"The hardening and tempering of steel is one of the world's oldest arts but, through the efforts of men such as Dr. Bain, it has evolved

Edgar Collins Bain

into one of our newest sciences. For hundreds of years, men have hardened steel by the well known process of heating and then rapidly quenching. It has been determined during the past three quarters of a century that this hardening operation involved a change in the crystalline form of the steel, and that hardening resulted from the lowering by rapid cooling of the temperature at which this change took place. It has remained, however, for Dr. Bain to show that the structure and, therefore, the hardness and mechanical properties of steel could be controlled by interrupting the cooling of the metal at predetermined temperature levels during quenching, maintaining it at such temperatures for a predetermined time and then permitting it to cool to room temperature. This has become a standard method of studying the mechanism of transformation of steels and other alloys as

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well. Applied commercially, this process is known as 'austempering.' A new micro-structure is developed, now recognized throughout the metallurgical profession as 'bainite.'

"I present Edgar Collins Bain, of Pittsburgh, Pa., as a candidate for a John Price Wetherill Medal, 'In recognition of his pioneer investigations into the structure of metallic alloys and particularly for his original work in the application of the isothermal method of studying the rates of transformation of austenite.'"

MR. NALLE: "Dr. Bain, by the same authority, I present to you a John Price Wetherill Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE CLAMER MEDAL

(To William Hume-Rothery, Royal Society Warren Research Lecturer, Metallurgical Chemistry, University of Oxford, England.)

MR. NALLE: "Mr. William B. Coleman is now recognized as the Sponsor of the Francis J. Clamer Medalist. Dr. W. A. Macfarlane, Scientific Attaché to the British Embassy in Washington, has come here this evening as proxy for Dr. Hume-Rothery."

William Hume-Rothery

MR. COLEMAN: "Mr. President, I present William Hume-Rothery for an award.

"When metallic elements are melted together they form alloys having characteristic crystalline forms which largely determine their commercial usefulness. What type of crystalline form results has, for

centuries, been attributed purely to chance. There have been no laws to guide the metallurgist, who has had to depend entirely on past experience or on his own cut and try experimentation. Dr. Hume-Rothery has discovered a rule which relates the ratio of electrons to atoms in many of the solid solution alloys of many alloy systems with the various types of crystal structure which are found in them. The successful application of his rule to the crystal forms in alloys has resulted in the use of the designation, 'Hume-Rothery Compounds,' in referring to them.

"I present William Hume-Rothery, of Oxford, England, as a candidate for the Francis J. Clamer Medal, 'In recognition of his brilliant work in scientifically determining and interpreting the structure and behavior of metallic equilibrium systems.'"

MR. NALLE: "Dr. Macfarlane, by the same authority, I take pleasure in presenting to Dr. Hume-Rothery, through you, the Francis J. Clamer Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE BALLANTINE MEDAL

(To Sergei A. Schelkunoff, Research Mathematician, Bell Telephone Laboratories, Murray Hill, N. J.)

MR. NALLE: "Dr. Albert F. Murray is Sponsor for our Ballantine Medalist. The Chair recognizes Dr. Murray."

DR. MURRAY: "Mr. President, I present Sergei A. Schelkunoff for an award.

"The ever-broadening field of radio engineering owed much of its progress into the newer, little-explored high frequency and microwave regions to our Medalist, a mathematical theorist.

"During the past twenty years that he has been at the Bell Telephone Laboratories, his work has pointed the way to many important communication developments. One of these, of interest to those who observe network television programs, concerns wave propagation in the coaxial cable.

"Dr. Schelkunoff did early theoretical work on the propagation of microwaves in wave guides. These guides later were used extensively in Radar installations during World War II. He made the interesting discovery of the so-called $TE_{0,1}$ mode of propagation, in which the attenuation of the guide decreases as the transmitted frequency increases. This discovery may have a bearing on long-distance guided communication in the future.

"Here are a few of the many theoretical problems on which our Medalist has worked: Antennas, both broadband and highly-directive as well as metal lens antennas; radiating horns; the study of wave propagation in the upper atmosphere.

"Dr. Schelkunoff, during World War II, acted as consultant on Wave Propagation at the U. S. Naval Station at San Diego. To his credit is an unusually long and impressive list of thirty-one publications. He is the author of three books.

"I present Sergei A. Schelkunoff, of New York, New York, as a candidate for the Stuart Ballantine Medal, 'In consideration of his out-

Sergei A. Schelkunoff

standing contributions to the extension of electromagnetic wave theory, particularly his mathematically-based concepts so helpful to the radio engineer.'"

MR. NALLE: "By the same authority, I present to you, Dr. Schelkunoff, the Stuart Ballantine Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANKLIN MEDAL

(To The Svedberg, Director, The Gustaf Werner Institute for Nuclear Chemistry, Upsala, Sweden.)

MR. NALLE: "The final Sponsor of the evening, Mr. C. H. Masland, II, is now recognized, to present the candidate for the Franklin Medal. Mr. Ingemar Hägglöf, Counselor of the Swedish Embassy in Washington, has come here this evening as proxy for Dr. Svedberg."

MR. MASLAND: "Mr. President, I have the honor to present as a candidate for the Franklin Medal The Svedberg, for many years Professor of Physical Chemistry at Upsala University and, since the first of September of this year, Director of the Gustaf Werner Institute for Nuclear Chemistry at Upsala, Sweden.

"Ever since the invention of the wheel, of the inclined plane and of the lever, the invention and development of tools have been among the most important of man's activities. Professor Svedberg has invented a new and very useful tool. He has developed it so that it performs the functions for which it was created and he has taught others to use it.

"Under the influence of the earth's gravitational field, particles suspended in a fluid settle. Particles of different masses settle at different rates. By observing this rate of sedimentation, it is possible to draw conclusions regarding the mass of the particles.

"As the size of particles becomes smaller, the rate of sedimentation becomes slower until, when we reach particles of the size of large molecules, the rate of sedimentation is so slow under the influence of the

The Svedberg

earth's gravitational field that other factors interfere with the observation of that rate of sedimentation. In order to secure more rapid sedimentation of particles of molecular size, Professor Svedberg turned to the centrifuge. We have centrifuges and we have super-centrifuges and Professor Svedberg has given us the ultracentrifuge. With these devices, Professor Svedberg has been able to develop centrifugal field intensities 900,000 times that of the earth's gravitational field and centrifugal field intensities 100,000 to 200,000 times that of the earth's gravitational field are in regular use for the purpose of determining molecular weights of molecules from the largest to the smallest, from the large protein molecules down to molecules as small as that of lithium chloride. With this device, Professor Svedberg has determined the weight average molecular weights of the very large molecules, particularly proteins in which scientists are so much interested these days.

This method is the only method in existence for determining such molecular weights.

"Mr. President, 'In consideration of his development of the ultracentrifuge, of his development of methods for use of the ultracentrifuge in determining molecular weights and the distribution of molecular weights, of high molecular weight proteins, and in consideration of his inspiring leadership of others in the field of colloid chemistry,' I take great pleasure in presenting The Svedberg."

MR. HÄGGLÖF: "Mr. President, Professor Svedberg has asked me to say how very much he regrets not being able to be here to-night. On his behalf, I wish to express his deep gratitude for the high distinction that the Franklin Institute has bestowed upon him.

"Professor Svedberg has lived and worked in the United States. Throughout his life he has maintained close contact with American scientists. He has in these ways acquired a profound knowledge of, and great admiration for, American science, scientists and scientific institutions. To have been honored by the distinguished Franklin Institute and to have received the Franklin Medal, the highest award for physical sciences in the United States, is therefore to him a very great satisfaction indeed.

"On behalf of Professor Svedberg and on behalf of Sweden, his country and mine, I wish to thank you most sincerely."

MR. NALLE: "Mr. Hägglöf, it is my pleasant privilege to present to Dr. The Svedberg, through you, this Franklin Medal with its accompanying Certificate and Report, togéther with a Certificate of Honorary Membership in The Franklin Institute."

"Dr. Allen, will you, as Secretary of The Institute, please assist Dr. Macfarlane and Mr. Hägglöf in any arrangement necessary to send these awards to their recipients in England and Sweden?"

DR. ALLEN: "Yes indeed, Mr. Nalle."

MR. NALLE: "It is now my pleasure to present Dr. J. Burton Nichols, of the duPont Experimental Station, who, as a personal friend and former student and colleague of Dr. Svedberg, will read to us a paper entitled "Giant Molecules in Solution," specially prepared for this evening's event by Dr. Svedberg.*

Mr. Nalle, after Dr. Nichols had read this paper, conveyed to him the thanks of the Institute, and dismissed the assembly with a cordial word and declared the meeting adjourned.

* See page 469 for "Giant Molecules in Solution."

MINUTES OF STATED MEETING OF THE FRANKLIN INSTITUTE

Wednesday, November 16, 1949

The November Stated Monthly Meeting of The Franklin Institute was held November 16, 1949 at 8:15 P.M. together with the Joint Lecture of the Philadelphia Section of the Illuminating Engineering Society. The meeting took place in the Lecture Hall with a capacity house in attendance.

President Nalle opened the short business session of the Stated meeting by announcing that the minutes of the October meeting appeared in the November issue of the JOURNAL OF THE FRANKLIN INSTITUTE and if there were no corrections they would stand approved as printed. There was no contrary motion. The President then called for a report from the Secretary.

In his report the Secretary announced the following additions to the membership for the month of October:

Active	42
Associate	45
Student	24

The total membership recorded as of the 30th of October was 5384.

Two announcements of interest to the membership were made by the Secretary. The first concerned the Christmas Show in the Planetarium which is scheduled to open to the public on November 29. All were urged to come and bring their friends to see "The Star of Bethlehem." The second announcement concerned the joint meeting of the Americal Society for Testing Materials and The Franklin Institute which is to be held on December 14, 1949. This will be a symposium on the subject "Research for Management" and many excellent speakers will be heard on this topic.

President Nalle then welcomed the officers and the members of the Philadelphia Section of the Illuminating Engineering Society to their first Joint Lecture Meeting at The Franklin Institute. He introduced Mr. Fred Pyle their Chairman, who in turn introduced Mr. Alston Rodgers, the speaker of the evening.

Mr. Rodgers, Senior Lighting Engineer of General Electric Lighting Institute in Cleveland, Ohio, chose for his topic "Light Sorcery." He presented an entertaining story of the science and research which made possible the lamps of today. He demonstrated a series of spectacular experiments with infra-red, ultra-violet and high-frequency radiations, and bewitched the audience with the mysterious "Ukelitely," which uses a glow lamp as a basis for a musical instrument.

HENRY B. ALLEN
Secretary

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS

ARCHITECTURE AND BUILDING

FRANKL, LEE. Home Repairs Made Easy. 1949.

ASTRONOMY

MAYALL, R. NEWTON AND MAYALL, MARGARET. Skyshooting; Hunting the Stars. 1949.

BIOGRAPHY

AMERICAN DENTAL ASSOCIATION. Horace Wells; Dentist, Father of Surgical Anesthesia. 1948.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Colloid Science. Volume 2. 1949.
FIERZ-DAVID, HANS EDWARD. Die Entwicklungsgeschichte der Chemie. 1945.
GAADE, W. Begenselen der Organisch-chemische Nomenclatuur. 1948.

DICTIONARIES

Blakiston's New Gould Medical Dictionary. 1949.

ELECTRICITY AND ELECTRICAL ENGINEERING

HINDE, D. W. AND HINDE, M. Electric and Diesel Electric Locomotives. 1948.

ELECTRONICS

LYTEL, ALLAN. TV Picture Projection and Enlargement. 1949.
GROB, BERNARD. Basic Television Principles and Servicing. 1949.

ENGINEERING

WILLIAMS, CLIFFORD DAVID AND HARRIS, ERNEST C. Structural Design in Metals. 1949.

MANUFACTURE

Steel Plates and Their Fabrication. 1947.

MATHEMATICS

BOCHNER, SALOMON AND CHANDRASEKHARAN, K. Fourier Transforms. 1949.
GOLDMAN, STANFORD. Transformation Calculus and Electrical Transients. 1949.
HOPF, L. Introduction to the Differential Equations of Physics. 1948.
JEFFREYS, HAROLD. Theory of Probability. Second Edition. 1949.
JOHNSON, LEE H. The Slide Rule. 1949.
SEMPLE, J. G. AND ROTH, L. Introduction to Algebraic Geometry. 1949.

MECHANICAL ENGINEERING

AMERICAN SOCIETY OF TOOL ENGINEERS. Tool Engineers Handbook. 1949.
GIRVIN, HARVEY F. Applied Mechanics. Second Edition. 1949.

METALLURGY

SCHULZE, R. BURT. Aluminum and Magnesium Design. 1949.

PHYSICS

CHALMERS, JOHN ALAN. Atmospheric Electricity. 1949.
JAKOB, MAX. Heat Transfer. Volume 1. 1949.
ROSSI, BRUNO B. AND STAUB, HANS HEINRICH. Ionization Chambers and Counters. 1949.
RUSHBOOKE, G. S. Introduction to Statistical Mechanics. 1949.

RADIO

FEDERAL TELEPHONE AND RADIO CORP. Reference Data for Radio Engineers. Third Edition. 1949.

SANITARY ENGINEERING

HARDENBERGH, WILLIAM A. Water Supply and Purification. Second Edition. 1949.

STEAM ENGINEERING

POTTER, PHILLIP J. Steam Power Plants. 1949.

WOOD

CORKHILL, THOMAS. A Glossary of Wood. 1948.

MUSEUM

Astronomy being the oldest of the sciences, it is inevitable that shreds of very ancient beliefs should still cling to its fringes in spite of all that astronomers can do to dislodge them. However, education has done much to free the human mind from the tyranny of superstitious faith in celestial omens and portents. Simple natural phenomena, such as a solar eclipse or the more rare appearance of a comet, could strike terror into a whole population. Our thinking has progressed beyond this state of ignorant superstition but fear of the future plays such an important role in people's lives that a conspicuous blot upon the intelligence of our contemporary thought is the widespread belief in astrology.

On the other hand, certain celestial phenomena, like the rainbow, have always been regarded as favorable omens, although their credibility was no greater than that of the evil omens, since all depended upon quite simple natural causes. The one celestial phenomenon which has retained its grip upon the imagination of multitudes is the Star of Bethlehem. Without denying that the appearance of this star might have had a supernatural origin, astronomers have submitted the question to a careful examination with a view to ascertaining whether or not there is a rational explanation for the appearance of some unusual celestial object at the time of the Nativity. Their investigations have brought to light some exceedingly interesting information relating to the errors which have crept into our accounting of time, and to the odd combinations in proximity which result from the planetary motions.

Each year during the month of December it is customary to devote the Planetarium demonstration to describing the various celestial phenomena which could have had an especial significance to the minds of those who saw the Star of Bethlehem. The extraordinary flexibility of the planetarium instrument enables the demonstrator to reproduce the skies as they appeared at the time in Palestine. This leads to an account of a rare conjunction of planets which occurred in the year 7 B.C. and to reasons why the possible date of the Saviour's birth should be post-dated to that year. The whole makes a story of absorbing interest.

The scientific aspect of the heavens at about the time of the Nativity having been adequately treated, the devotional aspect is then emphasized. Appropriate music and the ingenious applications of light and color are used to tell the story of the Nativity.

This annual demonstration has become a well established tradition of Philadelphia's Christmas week, and is attended by thousands of people who find this a most acceptable opportunity for recapturing the spirit of the season. Those who witness the demonstration for the first time almost invariably remark upon the impressive nature of the effort to combine the wisdom of today with the simple faith of the past in order to tell a complete story of an event, the spiritual significance of which has been and will continue to have an enduring influence.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

COULSON, THOMAS: The Franklin Institute from 1824 to 1949.

FANO, ROBERT M.: Theoretical Limitations on the Broadband Matching of Arbitrary Impedances.

FANO, ROBERT M.: A Note on the Solution of Certain Approximation Problems in Network Synthesis.
OUGHTON, CHARLES D.: Xerography.
MINAMIOZI, K. AND H. OKUBO: A Note on the Notch Effect of Metals.
SUSSKIND, CHARLES: Henry Cavendish, Electrician.
BOYAJIAN, A.: Origin and Meaning of Circular and Hyperbolic Functions in Electrical Engineering.
SILBERSTEIN, LUDVIG: Developable and Developed Silver Halide Grains.
ROOP, WENDELL P.: The Part of Octahedral Theory in the Study of the Plasticity of Metals.
HERSHBERGER, W. D. AND L. E. NORTON: Servo Theory Applied to Frequency Stabilization with Spectral Lines.
LEITNER, A. AND R. D. SPENCE: The Oblate Spheroidal Wave Functions.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, NOVEMBER 16, 1949

ACTIVE FAMILY

Alfred J. Cooke
Maurice J. Cooper
Allen A. Gale
Albert Greenberg, Sr.
Clyde H. Stephens

ACTIVE

Frederick H. Allen, M.D.
Alfred Arden
Royden Astley, M.D.
Ardrey M. Bounds
Walter E. Champlin
Frank Wade Conway
Raymond B. Crean
Louis Davis
William L. Davis
Bernard R. Horn
Joseph La Rocca, Jr.
William L. Lyon
Robert E. Mack
William F. Maxfield
Leo P. McManus
Walter Mulligan
W. A. Myers
J. W. Noll
Arnold Raines
Robert F. Robbins
Edward Semel
Lewis S. Somers
George B. Thomas

ACTIVE NON-RESIDENT

Richard Kroth
Robert McKinley, Jr.
Francis J. O'Brien
David E. Pearsall
Paul Wengraf

NECROLOGY

Harry W. Ellis '36
Charles L. Gabriel '45
Jonathan Godfrey '40
Sigmund S. Greenbaum, M.D. '48
George Walter Jarrett '46
W. J. Jeffries '42
Jack Kapp '48
Joseph A. Martocello '21
James H. McKee, M.D. '48
F. M. Murphy '43
John H. Packard '89
Ralph Pemberton, M.D. '24
Frank H. Stewart '20
Walter S. Thomson '36

BARTOL RESEARCH FOUNDATION

Deuteron Bombardment of N^{14} and C^{14}.*—E. L. HUDSPETH AND C. P. SWANN. Further bombardments of N^{14} have been made with 1.05 Mev deuterons. The arrangement was identical with that previously reported, except that the gas cell was covered with nickel rather than aluminum foil. This change was made when we found that the reaction $Al(d,n)$ was producing an objectionable background. Under these conditions, we find that the neutrons from $N^{14}(d,n)$ belong to only one group; a second group observed in this work probably arises from carbon contamination. It is therefore likely that $N^{14}(d,n)$ is a good source of monochromatic neutrons. Our results are in agreement with observations recently made in another laboratory,[1] but differ from earlier results.

The $C^{14}(d,n)$ reaction has also been observed with a target of $BaCO_3$ which was enriched about 40 per cent in C^{14}. The calculated Q-value for the formation of N^{15} in its ground state is 8.02 Mev, and we have observed neutrons corresponding to this case. Other groups, expected from consideration of the known excited levels of N^{15}, are also observed. The $C^{14}(d,n)$ reaction yields more than twice as many neutrons as $C^{13}(d,n)$ for deuterons of 1.26 Mev energy.

* Work in progress.

[1] W. M. GIBSON AND D. L. LIVESEY, *Proc. Phys. Soc.* (London), Vol. 60, p. 523 (1948).

Abstract of* An Investigation of the Disintegration Schemes of Ru^{103} and Rb^{86}.—C. E. MANDEVILLE AND E. SHAPIRO. The disintegration schemes of the 42-day Ru^{103} and the 19-day Rb^{86} have been studied with the use of absorption and coincidence methods. Ninety-two per cent of the disintegrations of Ru^{103} proceed by way of an inner beta ray group at 0.15 Mev coincident in time with a gamma ray of energy 0.52 Mev, which is (4 ± 1) per cent converted. A harder beta spectrum having a maximum energy of 0.68 Mev leads to the ground state of Rh^{103}.

It is estimated that (12 ± 2) per cent of the beta rays of Rb^{86} are contained in the inner spectrum. Absorption measurements confirm the energy values given by the Indiana group.

* As yet unpublished, but to appear in the *Physical Review*.

Unpublished Notes.—Dr. S. C. SNOWDON.

Ion Source

The possibility of using 10 cm. microwave radiation for the efficient production of protons and deuterons in the Van de Graaff generator is being investigated. Since a small region of the order of a cubic millimeter can be efficiently ionized with microwave energy, it was thought that the gas and power consumption of the source could be kept to a minimum by the use of 10 cm. CW microwave power. At present the CW magnetron supplying 100 watts and the R.F. voltage necessary for the Cockroft-Walton cascade rectifier (60 K.V.) are available. The oil insulated cascade rectifier and the vacuum system for testing have yet to be built.

Spectrometer

A large air-core double magnetic lens spectrometer for measuring beta-ray and gamma-ray energies is under consideration. The necessary copper and motor generator set with amplidyne control are available. A detailed theoretical investigation of the actual trajectory—accurate to the third order in the initial slope—of an electron in an immersion lens has been completed. The coil design should be able to handle quite high currents but, at present, we are keeping the power requirements down to 20 K.W. and expect to be able to focus three Mev beta-particles with high gathering power (1 to 10 per cent of a sphere) and with moderate resolution (1 to 3 per cent).

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

***Abstract of* A New Fourier Coefficient Harmonic Analyzer.**[1]—S. CHARP.[2] The Fourier coefficient harmonic analyzer is a device for evaluating the Fourier coefficients of periodic waves, or sections of complex waves which are assumed to be periodic outside of the interval within which the wave is to be analyzed. The machine (Fig. 1) can be considered to be a special-purpose differential analyzer, permanently interconnected to solve for the Fourier coefficients. Figure 2, a top front view of the analyzer with the protective cover removed,

[1] *AIEE* Paper 49-163, presented at the *AIEE* Summer General Meeting, Swampscott, Mass., June 22, 1949. *AIEE Proceedings*, Section T9163, Vol. 68 (1949).

[2] Senior Research Engineer, The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa. The harmonic analyzer was developed while the author was Research Associate in Electrical Engineering, Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pa.

shows the ball and roller type mechanical integrators which carry out the following integrations for the coefficients:

$$A_0 = \frac{1}{\pi}\int^{2\pi} F(\theta)d\theta, \tag{1}$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi} F(\theta)\cos n\theta d\theta, \tag{2}$$

and

$$B_n = \frac{1}{\pi}\int_0^{2\pi} F(\theta)\sin n\theta d\theta, \tag{3}$$

$$F(\theta) = A_0/2 + \sum_1^{\infty} A_n \cos n\theta + \sum_1^{\infty} B_n \sin n\theta. \tag{4}$$

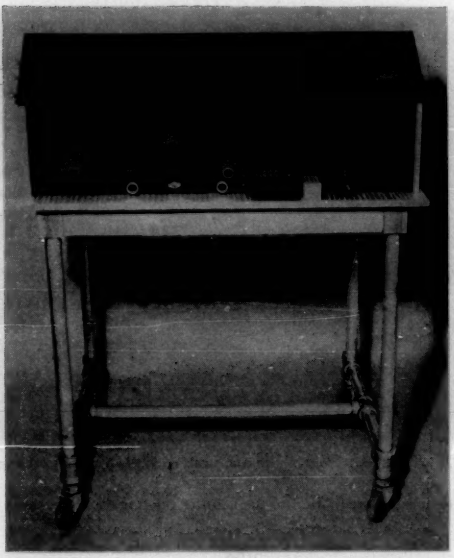

FIG. 1. Over-all view of the harmonic analyzer.

The function, $F(\theta)$, is the wave whose periodical components are desired. In accordance with the theory of the Fourier series, it is also assumed to be given by the series in Eq. 4.

The coefficient A_0 is developed by an integrator whose incremental output is given by

$$dA_0 \propto F(\theta)d\theta. \tag{5}$$

This relationship can be used in Eq. 2 and 3 resulting in:

$$A_n = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} \cos n\theta d\left[\int F(\theta)d\theta\right] = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} \cos n\theta dA_0, \tag{6}$$

and

$$B_n = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} \sin n\theta d\left[\int F(\theta)d\theta\right] = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} \sin n\theta dA_0 \tag{7}$$

With Eq. 1, these equations are much more suitable for mechanical solution. Since the sine and cosine coefficients of the Fourier series are developed in this machine, the magnitude and phase of each frequency component can be determined by the following:

$$C_n = [A_n^2 + B_n^2]^{\frac{1}{2}}, \tag{8}$$

and

$$\phi_n = \tan^{-1}(B_n/A_n). \tag{9}$$

Four ball and roller integrators are used in the analyzer; three to perform the integrations called for in Eqs. 1, 6, and 7, and one to serve as a variable speed drive to adjust for frequency, obviating the use of change gears. The outputs of the integrators are individually recorded on counters, located at the front upper left hand part of the analyzer These can be read to 0.02 revolution

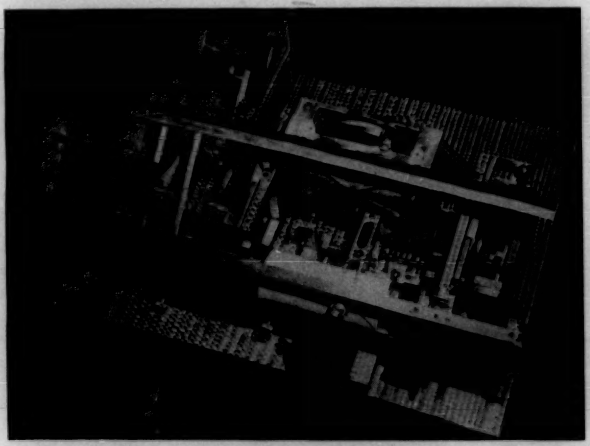

FIG. 2. Top front view of the analyzer with plastic cover removed.

The Fourier coefficients are related to the counter readings as follows:

$$A_0 = \frac{\text{Counter Reading}}{576K} + 2f_{avg}, \tag{10}$$

$$A_n = \text{Counter Reading}/576K, \tag{11}$$

and

$$B_n = \text{Counter Reading}/576K. \tag{12}$$

Here,

$K = \dfrac{5}{[f_{max} - f_{min}]}$ = conversion factor to change the physical measured data to inches displacement along the plotted curve,

f_{avg} = arithmetic mean of the peak to peak values of the function $F(\theta)$ to be analyzed,

f_{max} = maximum value of the function $F(\theta)$, and

f_{min} = minimum value of the function $F(\theta)$.

The curve to be analyzed is plotted on a piece of paper to a size not greater than 5×10 in. It is then wrapped around the plastic cylinder shown in Fig. 2 and inserted in the machine.

When the machine is in operation, the cylinder rotates an amount proportional to the independent variable. The operator tracks the curve by sighting through an eyepiece and turning the tracking handwheel located on the front panel. This operation introduces the function $F(\theta)$ into the integrator which solves for Eq. 1, the output of which is the differential input to the integrators for Eqs. 6 and 7. The sine and cosine integrands of these latter equations are developed by a Scotch-yoke mechanism.

Each time the curve is scanned the Fourier coefficients for that harmonic appear on the counters. The frequency is altered by adjusting the variable speed-drive to the Scotch-yoke.

Since the completion of the machine it has been applied to solving problems in many engineering fields including studies in servo-mechansims, crystal structure periodicities, radar tracking of airplanes, and others.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

Seminar on Pathological Sera.—Dr. Kai O. Pedersen of the Institute of Physical Chemistry of the University of Upsala, Sweden, addressed the Seminar of the Biochemical Research Foundation on Wednesday afternoon, June first.

For a number of years Dr. Pedersen has utilized the Svedberg ultracentrifuge and the Tiselius electrophoresis apparatus to examine pathological sera furnished by Dr. Jan Waldenström of the University Medical Clinic. Dr. Waldenström has supplemented the ultracentrifugal and electrophoretic analyses by salt fractionation and Kjeldahl analyses, by measurements of specific gravity, refractive index, and relative viscosity at 13° and 37° C., and by the formol gel, cadmium, and alcacid tests. The more recent electrophoretic work has been carried out in collaboration with Dr. Niels Harboe of the University of Copenhagen.

In the ultracentrifuge diagram of normal human serum, the albumin is found in the boundary with a sedimentation constant of 4.5 S. The alpha globulin is divided between the 4.5 S and 20 S boundaries, and the beta globulin among the 4.5 S, 7 S, and X boundaries; the gamma globulin appears almost always to sediment with the 7 S boundary.

Many pathological sera are characterized by hyperglobulinemia. In severe burns and in acute infections accompanied by tissue destruction (acute tuberculosis, pneumonia, cancer with metastases) there is an increase of alpha globulin; the globulin thus formed sediments with the 4.5 S boundary. In certain renal diseases there is an increase of the alpha and beta components in electrophoresis, and of the 4.5 S component in the ultracentrifuge. In most cases where there is an increase in the total plasma protein, however, the augmentation is due to an increase in the gamma globulin, and this is usually correlated with an increase in the 7 S component.

In some cases of myeloma there may be an increase of one of the slow moving beta components; this is correlated either with an increase in the 7 S component or with the appearance of a new component of sedimentation constant 11 S. In a few rare cases of essential macroglobulinemia (Waldenström, *J., Acta Med. Scand.*, 117, 216 (1944)), a very high concentration of the 20 S component has been found. This is associated with the beta or gamma globulin in electrophoresis. It always gives rise to a sharp and well-defined component both in electrophoresis and in the salting-out procedure of Derrien (*Svensk. Kem. Tid.*, **59**, 139 (1947)). The molecular weight is about one million. Augmentation of this component confers certain abnormal characteristics on the

serum: high relative viscosity with a negative temperature coefficient, leading to a viscosity index, $100 \cdot \eta_{13°}/\eta_{37°}$ (Waldenström, *loc. cit.*), above the normal value of 100; high euglobulin content; and rapid formol gel, cadmium, and alcacid tests.

Dr. Pedersen and Dr. Waldenström in collaboration with Dr. Sonck of Helsingfors have chosen lymphogranuloma venereum, a compara-

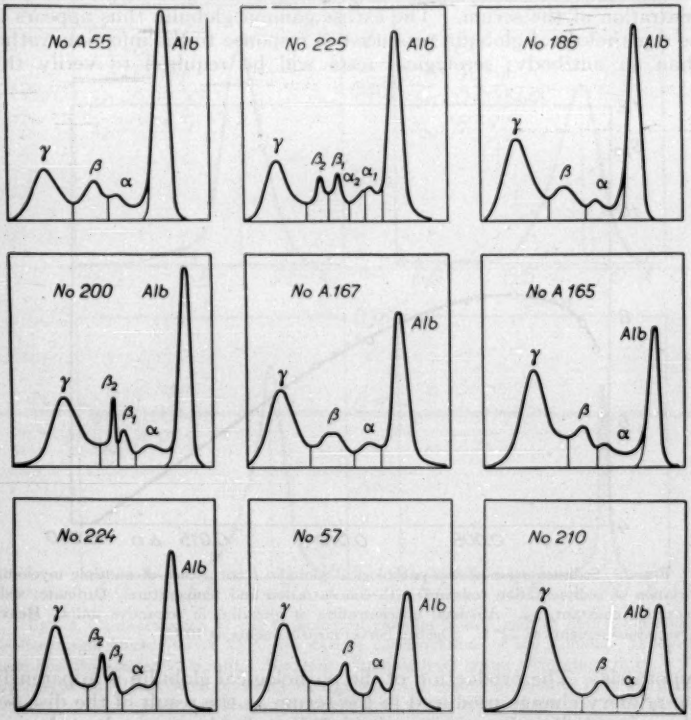

FIG. 1. Electrophoretic diagrams from nine cases of lymphogranuloma venereum with varying hyperglobulinemia.

tively rare disease in Sweden, as a type of chronic virus infection. In the serum of patients with this disease (Fig. 1) the globulin concentrations are high and the albumin concentrations low. The increased globulin is associated with the gamma globulin, and forms a very broad boundary; the sedimentation constant is 7 S. The relative

viscosity of the serum parallels the gamma globulin content and exhibits no pathological temperature coefficient. The formol gel and other tests are rapid. When separated by salt fractionation methods, the gamma globulin from lymphogranuloma venereum is apparently inhomogeneous with respect to both charge and mass.

Successful treatment of the disease by sulfa drugs or other therapeutic measures is accompanied by a drop in the gamma globulin concentration of the serum. The excess gamma globulin thus appears to be a pathological globulin produced in response to the infection, rather than an antibody; serological tests will be required to verify this

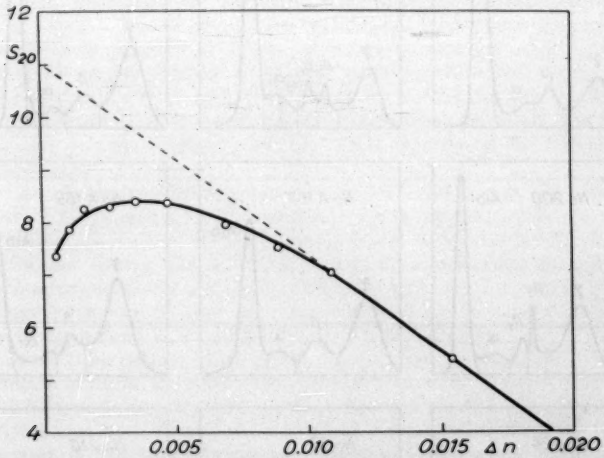

FIG. 2. Sedimentation of the pathological globulin from a case of multiple myeloma. Variation of sedimentation constant with concentration and temperature. Ordinate: sedimentation constant, s_{20}. Abscissa: concentration of globulin, in refractive units. Heavy curve: measurements at 22° C. Dashed curve: measurements at 10°C.

hypothesis. The production of the pathological globulin is apparently the primary change produced in the serum as the result of the disease. The reduced albumin concentration is believed to be a secondary change, for the extent of the reduction appears to vary with, and possibly to be determined by, the concentration of pathological globulin (see Fig. 1).

Occasional instances of hyperglobulinemia with increased gamma globulin such as characterizes lymphogranuloma venereum have been found with no apparent pathological cause. The condition might reasonably be the result of chronic infection by a non-pathogenic virus; it has not, however, responded to treatment with sulfa drugs. Hyper-

globulinemia with increased gamma globulin has also been found in liver cirrhosis.

A different type of pathological globulin is produced in myeloma. One sample of myelomatous serum contained 58 per cent of beta$_2$ globulin; the pathological globulin could be separated as a homogeneous

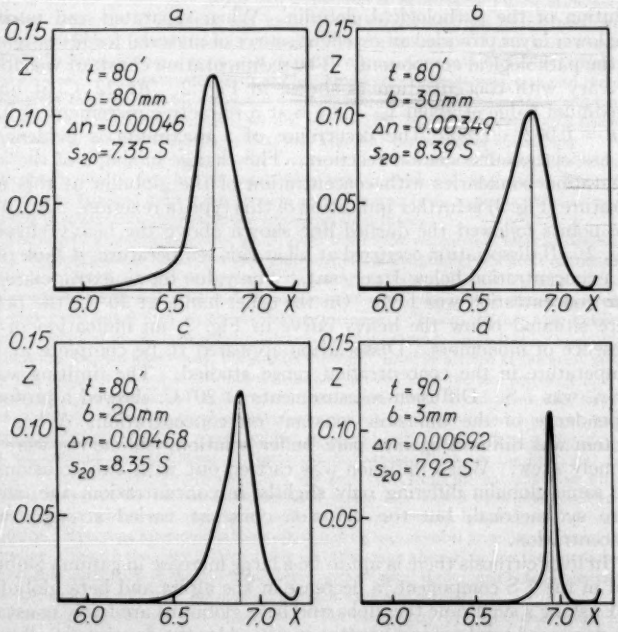

FIG. 3. Sedimentation of the pathological globulin from a case of multiple myeloma Sedimentation boundaries at 22° C. for various concentrations of the globulin. Ordinates: scale line displacements, in mm. Abscissae: distances from center of rotation, in cm. The symbols in each block have the following significance: t = time in minutes from the attainment of full rotor speed; b = distance from the scale to the middle of the cell; Δn = globulin concentration in refractive units; s_{20} = sedimentation constant.

component by salt fractionation methods (Derrien, Y., *loc. cit.*). Other samples of myeloma serum contained much gamma globulin, whereas both alpha and beta globulins were normal. In each instance the pathological component gave a sharp boundary on electrophoresis and a homogeneous component on salt precipitation. The salt precipitation ranges varied, however, with the mobility.

A patient in the early stages of myeloma showed a moderate increase of gamma globulin, with little change in the other components of the serum. Three years later the same patient, in the terminal stages, showed a large increase of gamma globulin.

A sample of myeloma serum from Holland separated spontaneously at 2° C. into two layers, the lower of which proved to be a concentrated solution of the pathological globulin. When separated and purified, the lower layer provided an excellent source of material for investigation of the pathological component. The sedimentation constant was found to vary with concentration as shown in Fig. 2. At 22° C. it had a maximum value of about $s_{20} = 8.5$ S at a refractive increment of 0.007 ($\Delta n = 0.002 \sim 1\%$). The occurrence of a maximum is evidence of an association-dissociation reaction. The change in shape of the sedimentation boundaries with concentration of the globulin at this temperature (Fig. 3) is further indication of this type of reaction. At 10° C. the points followed the dashed line shown above the heavy curve in Fig. 2. If dissociation occurred at all at this temperature, it took place at a concentration below 1 per cent. The value for s_{20} extrapolated to zero concentration was 11 S. On the other hand, at 30° C. the points were situated below the heavy curve in Fig. 2, an indication of the presence of monomers. Dissociation appeared to be complete at this temperature in the concentration range studied. The limiting value for s_{20} was 7 S. Diffusion measurements at 20° C. showed a profound dependence of the diffusion constant on concentration. When the protein was diffused against pure buffer solution, the curves were extremely skew. When diffusion was carried out with two solutions of the same globulin differing only slightly in concentration, the curves were symmetrical, but the diffusion constant varied strongly with concentration.

In liver cirrhosis there is apt to be a large increase in gamma globulin and in the 7 S component, a decrease in the alpha and beta globulins. In Cushing's syndrome the alpha and beta globulins are fairly constant; the decreased total protein content is caused by the decrease in albumin and gamma globulin. A typhoid patient with complete loss of appetite showed an extremely low content of albumin and of total protein, with almost normal values for the alpha and beta globulins. With return of appetite the patient recovered, and recovery was paralleled by return of the albumin content to normal.

It is apparent that the hyperglobulinemia which characterizes many diseases can manifest itself in numerous ways. The results so far obtained have encouraged the investigators in their hope that physicochemical methods may give a clearer insight into protein derangement than has hitherto been possible.

V.2
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BOOK REVIEWS

MAX PLANCK'S SCIENTIFIC AUTOBIOGRAPHY AND OTHER PAPERS, translated from the German by Frank Gaynor. 192 pages, 14 × 22 cm. New York, The Philosophical Library, 1949. Price, $3.75.

It is hard to believe that almost half a century has passed since Planck made his first disclosure of the law of radiation and its deduction which raised him to the front rank of physicists of all time. He was not a prolific writer but he was certain of an interested audience whenever he chose to write or speak, and this small volume of essays is the pendant which completes his work. The essays deal with problems which, for many decades, occupied the mind of this great thinker, such as:—true and fictitious issues in science, the meaning and limit of exact science, physics and causality, natural science and the religious world. All belong to the period near the end of his life, so that they represent the essence of his teaching.

There is nothing in the volume which ranks with the two papers Planck presented before the Berlin Physical Society in 1900, establishing him as a thinker of the foremost order. Such was not to be expected for it is not given to a single human being to produce two conceptions equalling in importance the conception of the quantum theory.

The autobiographical fragment which gives title to the volume is of especial significance. In the simplest possible terms and with great charm the living mind of a great contemporary is unfolded. It tells how the first papers of the young student were not even read by the famous men at whose feet he sat. Helmholtz and Kirchhoff, under whom he studied, so ignored him that they contributed little to his mental advancement. Clausius did not even answer his letters. Rebuffed by the men who would have found it easiest to appreciate his work, Planck continued in his own lonely way, obeying an inner call, until he arrived at the great problem for which the path he had taken without guidance proved to be the best preparation. Thus he was able to recognize and to formulate, from measurement of radiations, the law which bears and immortalizes his name. Acceptance was not achieved without a struggle and the experiences through which he passed led him to the sad reflection that "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it."

This sketch of the mental development of a man who looked upon the search for the absolute as the noblest and most worth while task of the scientist is a document of touching simplicity. Everyone interested in the evolution of a great physical principle will benefit from its reading, although the story is carried only up to the stage where Bohr laid the foundation for a reasonable unification of the quantum theory with classical theory, and Schrödinger created wave mechanics and thereby the dualism between wave and particle.

Planck's long life was not all serenity. The controversy over his first original ideas, waged by Boltzmann, the atomist, against Ernst Mach, Ostwald, and the Energetics engendered no small amount of heat and irritation, but it is graciously overlooked in this retrospective view. Nor will any allusion be found to the bitter personal tragedies of his life, the death in action of his soldier son, and the stunning loss of his second son in the reign of terror which followed upon the attempt on Hitler's life in 1945. These were terrible blows to a man whose private life would otherwise have been as beautiful and contented as any man could wish for. The aloofness of his mind from personal sorrows is reflected in the omission to all allusions which would have caused Max Planck's recollections to be tinged with bitterness and disappointment. His admirers and his biographers will find this all too brief outline of his intellectual development a valuable work, not on any account to be overlooked.

T. C.

BASIC ELECTRONICS, by Royce G. Kloeffler and Maurice W. Horrell. 435 pages, 15 × 23 cm., drawings and photographs. New York, John Wiley & Sons, Inc., 1949. Price, $5.00.

"Basic Electronics" is less basic than it is comprehensive. The authors have attempted to introduce the entire field of electronics emphasizing the multitude of operations that can be

performed within this field, and they have done this very skilfully. Examples of every type of electron tube from the proximity fuse sub-miniature to the cavity magnetron used in pulse radar are discussed. Electronic apparatus ranging from the simple photocell exposure meter to the electron microscope and the G. E. betatron is described.

One of the most important points in justification of this book is its up-to-dateness, for it includes such recent developments as the Bell Telephone Laboratories' transistor and R. C. A.'s triode mechano-electronic transducer. It also includes the large variety of devices such as germanium and silicon crystals developed for radar and other purposes during the War.

Nearly every page of the volume contains some sort of illustration. In most cases these are drawings prepared to illustrate a teaching point discussed in the text; there are few vague apparatus photographs so often found in this type of non-mathematical treatment. Although the book is primarily descriptive, the use of these well-drawn, line illustrations and a simple choice of words convey a large amount of information.

The authors state in the preface that they intend this as a first course in electronics. The reviewer feels that the book provides material adequate for a first course in electronics for the liberal education of electrical power majors, general engineering students and those who do not intend to specialize in electronics. However, the rigorous preparation required for professional performance in electronics and communications indicates that a more thorough grounding in the physics of fundamental electrical phenomena would be a more profitable starting point than this general survey. The volume is useful for teaching purposes but has little value as a reference.

C. W. HARGENS

CHYMIA, ANNUAL STUDIES IN THE HISTORY OF CHEMISTRY, Tenney L. Davis, Editor in Chief. Volume 2, 143 pages, illustrations, 15 × 24 cm. Philadelphia, University of Pennsylvania Press, 1949. Price, $4.00.

It is a pleasure to record the appearance of the second volume of this interesting series. However, one regrets the untimely death of Mr. Davis, for as editor of these first two volumes he has builded well. In the present volume he has succeeded admirably in emphasizing the international character of the work, for more than half of the twelve articles represent the work of residents of five foreign countries: Canada, France, Germany, India, and the Netherlands. Such variety of nationality in authorship almost automatically insures the variety of subject matter which is a desirable feature of such a volume.

The lead article is by H. S. van Klooster and treats of the beginning of laboratory instruction in chemistry in the United States. After reviewing the early teaching of chemistry in this country, the author discusses the work of Amos Eaton, the first teacher of practical chemistry in America. The subsequent introduction of laboratory teaching into other colleges than that of Rensselaer Polytechnic Institute is sketched briefly.

Wilhelm Prandtl offers an interesting account of the chemical laboratory at the Bayerische Akademie der Wissenschaften in Munich. In the number and caliber of the teachers and industrial chemists who have been trained there, it holds an outstanding position. The article is well illustrated with portraits of the laboratory's leaders and of its buildings, unfortunately destroyed in the last war.

Individual studies of several figures in chemical history are given by W. Miles on Sir Kenelm Digby; by D. Reilly on William J. MacNeven, an Irish-American chemist, 1763–1841; and by R. J. Forbes on Sir Isaac Newton and his interest in chemistry.

Personal reminiscence is contributed by Edmund P. Hillpern who writes of some personal qualities of Wilhelm Ostwald. As a former assistant of Ostwald, Hillpern is able to furnish several interesting sidelights on the former's beliefs and habits.

R. Hooykaas discusses the experimental origin of chemical atomic and molecular theory before Boyle, and Mr. Davis, the editor, has contributed a well documented paper entitled "Pulvis fulminans" in which he discusses the various references to fulminating power. Other articles treat of some seventeenth century chemists and alchemists of Lorraine, the discovery of catalysis, by Désormes and Clément, the history of ambergris in India, and burning glasses.

With its wide variety of articles and well chosen illustrations attractively presented,

Chymia should offer something of interest to every student of chemistry. To facilitate its use the editors have added a name index of the first two volumes, further emphasizing its permanent reference value.

G. E. PETTENGILL

THE SPECTROSCOPIC PROPERTIES OF URANIUM COMPOUNDS, by G. H. Dieke and A. B. F. Duncan. Volume 2, Division III, National Nuclear Energy Series, 290 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., 1949. Price, $2.75.

This book describes in a rather complete manner the spectra of a large number of uranium compounds. It is one of a series, whose purpose is to record results of research done under the Manhattan Project and the Atomic Energy Commission.

The descriptive part of the book deals mainly with experimental work which has taken place at several different laboratories. The spectroscopic work was done at Columbia and The Johns Hopkins Universities. The investigation of crystal structure was carried out at Cornell University.

The book is divided into two parts. Part one deals with the spectroscopic properties of uranium compounds, while part two discusses their preparation. This is followed by three appendices and a bibliography.

The first chapter of Part I discusses experimental procedures. The instruments used are described. The preparation of samples and the methods used for low temperature studies are also discussed. The chapters following discuss the crystal structure and the general features of the spectra of uranyl compounds. This is followed by a more detailed discussion of their fluorescence and absorption spectra. Other chapter headings are "Intensity Measurements," "Miscellaneous Observations" and "Miscellaneous Uranium Compounds."

The second part begins with the chapters on the synthesis and preparation of uranyl compounds. There is a chapter which discusses the methods used in the study of crystal growth. This part is concluded with a summary of the whole work.

The book contains as part of the appendix about eighty-five pages of tables giving wavelength measurements and other spectral data for many uranium compounds.

An extensive bibliography is included which contains references not only to the spectroscopic and optical properties of uranium compounds but also to other properties such as photochemistry, color, and crystal structure. This bibliography has the references arranged in chronological order beginning with the famous papers of Nichols and Howes which were published in 1919.

This book deals with a highly specialized topic and consequently may not be of direct interest to a large number of scientists. It does, however, contain a great deal of information and should prove very valuable to persons working in the field of spectroscopy, especially those who are concerned with the compounds discussed.

The results of the research performed by a large number of people are recorded in this volume. It therefore represents an important contribution to our total of scientific knowledge.

MARSHALL D. EARLE

CONSTRUCTIVE USES OF ATOMIC ENERGY, edited by S. C. Rothman. 258 pages, plates, 14 × 21 cm. New York, Harper & Brothers, 1949. Price, $3.00.

Mr. Rothman has brought together fourteen articles by scientists in various fields, in an attempt to produce a book which would point up the potentialities of atomic energy for other than military purposes. The articles were, for the most part, written originally in technical journals and will not be well understood by the layman. Furthermore, this method of assembling a book leads to an ultimate product which is not a book as a unit, but is merely fourteen papers bound together.

The book shows clearly that the constructive aspects of atomic energy lie for the most part in research fields. The large industrial applications of atomic power for industry and atomic engines for aircraft are, as of July and August, 1948, applications for the future beset with many difficult problems. As for the uses in medicine, the article by Dr. C. P. Rhoads does a good job of making clear that only in rare cases can diseases be cured with atomic

radiations, and he feels that the important role of radioactive isotopes in medicine lies in the research studies they make possible, rather than in direct therapeutic application.

Mr. Rothman, in his preface, states that the articles included in the book were chosen in an effort to find material which would dispel the common misconception that atomic energy is beyond the comprehension of the layman. I agree with him that this is a common misconception, but I do not feel that this book does much to dispel it.

E. SHAPIRO

FREEZE-DRYING, by Earl W. Flosdorf. 280 pages, illustrations, 15 × 23 cm. New York, Reinhold Publishing Corp., 1949. Price, $5.00.

This is the first book entirely devoted to the subject of freeze-drying, a technique of drying material from the frozen state now being used for the preservation of antibiotics, foods, blood plasma and bacterial and viral cultures. The author has done much of the pioneer work on freeze-drying and is well qualified to present both the theoretical and the practical aspects of freeze-drying on either laboratory or industrial scale.

One portion of this book is devoted to the basic principles of physical chemistry on which the technique depends and which govern the design and operation of equipment. Much of the remainder of the book is devoted to the practical application of freeze-drying in research and industry, with particular attention to the conditions under which drying is carried out, and the equipment used in the commercial processing of medical and food products. A complete bibliography, a list of patents in this field, and an adequate index make this book doubly valuable to anyone who is interested in the preservation of labile materials.

LOUIS DESPAIN SMITH

TRANSFORMATION CALCULUS AND ELECTRICAL TRANSIENTS, by Stanford Goldman. 439 pages, 14 × 22 cm., illustrations and tables. New York, Prentice-Hall, Inc., 1949. Price, $6.25.

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Research and development workers in electrical engineering will find this book to be of great value as an aid in the analysis of transient electrical phenomena. Although practically no results are presented that have not appeared in other books, something new has been created here in the form of a consolidation and correlation of divers subjects in the field of applied mathematics as they pertain to transient-analysis problems. In this book one finds such seemingly unrelated mathematical topics as determinants, theory of functions of a complex variable, gamma and error functions, solutions in series, Bessel functions, etc. welded into a single cohesive bundle and clearly presented in direct relationship to transient problems in which they are useful. This is a coordinated, comprehensive treatment that has been needed in this field and which penetrates into an old subject with a modern, expanded viewpoint.

The principal aim of this book is to develop up-to-date methods of the Laplace transformation and its inverse as applied to electrical networks. In accomplishing this aim certain mathematical topics are introduced and used in numerous specific examples. Non-linear circuits are not treated nor is the problem of synthesis considered. Because the mathematical material is also useful in other phases of electrical engineering, the reader will strengthen his general background as well as aquire a familiarization with the tools of transient analysis.

Those individuals to whom the mathematics of this book is relatively unfamiliar will experience great difficulty if they attempt to study this book alone, despite its clarity. It deals largely with the important highlights of the subject rather than a thorough inquiry into each separate topic. This feature is apparently recognized, and partially compensated for, by excessive use of footnotes (over 230 of them!) many of which refer to other texts where a thorough treatment of the subject under discussion may be found. However, one wonders why an author index or bibliography has not been included, particularly because this is in the nature of a "consolidation" book. Perhaps it is because of the abundance of references to the author's previous book, *Frequency Analysis, Modulation and Noise*, which, despite its frequent "plugging," is a very worthwhile volume.

A significant weakness of this book is rooted in Dr. Goldman's attempt to bring moderately advanced material within the reach of college seniors. Certain topics are ridiculously simple

by contrast with the meaty middle and latter portions of the book. For instance, Chapter 1, "Determinants," contains material that can be found in many elementary mathematics books. This exceedingly elementary chapter contains some examples of the use of determinants in the solution of network problems which may be of some value, but the phasor diagrams and analytical geometry applications seem rather inappropriate. Similarly, a nineteen-page appendix "Basic Theory of Transmission Lines in the Steady State" (material that can be found in at least a half-dozen textbooks for undergraduates) seems far too long and unrelated to be justified by the very excellent and basic Chapter 10, "Transients in Transmission Lines." Another example is the author's failure to demonstrate the disadvantages of the classical method of analysis which is certainly a very desirable and logical way of introducing the novice to the more powerful and convenient Laplace transformation method. There are other examples of this divergence in which it seems that the two extremes are separated too far in level.

The net result is that many of these pages are essentially wasted. A senior in a high-level college would be able to cope with the first 128 pages but would tread on dangerous ground in proceeding further without a lengthy digression to several other works for fundamental background which is considered by the reviewer to be insufficient in this book for a first introduction to the subject. On the other hand, a person who is qualified to use this book to full advantage as a good centralized source of judiciously-selected mathematical information will simply ignore much of the elementary material. It would seem much better to have eliminated the latter, replacing it with a more complete treatment of the advanced aspects.

ERNEST FRANK

HEAT TRANSFER, Vol. I, by Max Jakob. 758 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1949. Price, $12.00.

This book is a comprehensive product of the author's many years of activity in research and teaching elementary, intermediate and graduate courses on the subject of heat transfer, and serves both as a treatise and as a textbook. Its main purpose is to show the development of ideas which led to the present state of knowledge on the subject. The main contents of this book, including the most important derivations, are presented in such detail that study of the original papers will not be necessary except in very rare cases. The book is international in scope, with particularly thorough coverage of extensive German literature.

Since the interrelation and peculiarities of conduction, convection and radiation become more evident in the parallel treatment, they are not dealt with separately. Accordingly, the first part of the book contains the basic equations of all three brancehs of heat transfer; the second part deals with the most characteristic properties occurring in each of these branches; then the solutions of the equations, with all their consequences, are shown in the conventional separation into special parts for each of the three kinds of heat flow as follows: Heat Conduction in Simple Bodies; Heat Transfer without Change of Phase or Constitution; and Heat Transfer Including Changes of Phase. Each of these parts contains straight solutions, similarity treatments, techniques, graphical methods, experimental methods, etc. The treatment of heat radiation in spaces of simple configuration, and selected fields for application of heat transfer will be presented in Volume II, now in preparation.

Appendices contain sample problems to be solved, a list of the symbols most often needed, conversion factors, bibliography and author index and a subject index.

This book, like the courses of the author, should serve the needs of mechanical, chemical, electrical, civil, and aerodynamic engineers, and physicists alike. The presentation is, therefore, an arbitrary compromise between the highly mathematical procedure in most technical publications and the presentation of empirical facts in others. For the benefit of readers with only a modest knowledge of higher mathematics, the mathematical procedures are explained in the first chapters and parts so that they may become acquainted with such typical features as finding and substituting suitable boundary conditions. The empirical material has been distributed throughout the text and presented wherever the analysis required it for illustration, verification or for showing the deficiencies of the theoretical analysis.

SAMUEL M. BERKOWITZ

TABLES AZÉOTROPIQUES, by Maurice Lecat. Tome premier, Azéotropes binaires orthobares. Deuxieme edition, 406 pages, 16 × 26 cm. Bruxelles, Chez L'Auteur, 1949. Price, 1000 Belgian francs.

In 1918 there appeared M. Lecat's first book on azeotropy which included some elementary considerations on the subject, tables of azeotropes, and a bibliography. Since that time he has had published some ninety works on the subject including both books and periodical articles. The bibliography which appeared in the 1918 volume has since been revised and expanded, and has been issued in two volumes, one in 1932 and the other in 1942.

The present title represents a revision or second edition of the tables of the 1918 work. Those tables were divided into four parts: binary systems with minimum boiling point; binary systems with maximum boiling point; ternary systems and quaternary systems. There were 2200 binary systems included and 250 ternary systems. The present volume is limited to binary systems and includes 6287 azeotropes and 7003 zeotropes, nearly all determined under 760 mm. of pressure. The forthcoming second volume will treat of the ternary systems as well as presenting M. Lecat's discussion of the empirical laws which may be set forth.

The tables are arranged by groups alphabetically from *Acides et Acides-cétones* to *Varia et Varia*, passing through, for example, *Acides et Oxydes*, *Cétones et Nitrodérivés*, and *Esters et Phénols*. The tables include the boiling point of each constituent, their difference, the boiling point of the azeotrope and its deviation, and the percentage of the first constituent in the mixture.

Bibliographical reference to the original sources is made by number to the entries in the two volumes of bibliography previously published, and in the case of M. Lecat's own work, to a list included in the present volume. References later than those included in the bibliography are given in full. An index by author and an index of the constituents of systems have been included.

In the forty odd years in which M. Lecat has been studying azeotropes, there has been a great increase in the knowledge of these mixtures and in their commercial importance. That much of this increase is due to M. Lecat will be recognized by all students of the subject and they will be grateful to him for providing this up-to-date comprehensive table of azeotropic values.

G. E. PETTENGILL

ALTERNATING CURRENT MACHINERY, by L. V. Bewley. 376 pages, illustrations, 16 × 24 cm. New York, The Macmillan Co., 1949. Price, $5.25.

Professor Bewley, in writing this new book on alternating current machinery for junior and senior students in electrical engineering, no doubt kept in mind the order in which most newly graduated engineers encounter the most important types of electrical machinery. The book introduces the general principles of magnetic circuits, followed by transformer theory. The basic design principles pertinent to rotating machines precede detailed treatments of specific machines, again in the approximate order in which these are encountered, namely: polyphase induction motors; single-phase induction motors; alternating current commutator machines; synchronous generators and motors; and last, synchronous converters.

What is different in this book? The author has approached the writing of this text with a fresh and long needed point of view, based upon his many years of industrial engineering and educational experience. For a long time, inadequate books on alternating current machinery have adhered to concepts and methods of analysis which should long ago have been abandoned because of their restricted and narrow approach. Here the author has attempted to unify the underlying theories of electrical machinery by introducing general methods of analysis which hold for many kinds of rotating machines. Several new concepts are introduced in the book which simplify an understanding of machinery theory.

The section devoted to transformers departs from the traditional treatment and assumes: (1) that all coils on the iron structure are independent, with interconnections to be made as desired for the transformer configurations to be studied; and (2) that the presence of a fictitious

short circuited coil can account for core loss. The use of differential equations greatly simplifies the presentation of the material.

Since most alternating current rotating machines have coil structures which have basic designs in common, a realization of the importance of the many empirical and computed design reduction factors is called for. This is essential to a full understanding of synchronous machines.

In an attempt to generalize the machine equations, the author has succeeded in developing all-inclusive basic equations for voltage generation and armature reaction which hold for a-c. or d-c. machines. A unification of concept naturally follows since one kind of machine differs from another only in the presence or absence of specific terms.

Many such examples of essential departures from the traditional treatment of machinery theory appear in the book. To review them in detail with an evaluation of their merit would not be proper in this resumé.

The many worked out examples and problems included in each chapter make the text ideal for a two semester course, to be supplemented by courses in electro-magnetic field analysis applied to magnetic structures. The one fault which this reviewer finds is the almost complete lack of references to the technical literature. This oversight should be corrected in future editions to make the book more desirable as a reference work not only for electrical engineering students but for the practicing engineer who desires a fresh approach in a review of basic a-c. machinery.

S. CHARP

TWO LECTURES. I. PRESENT SITUATION IN THE THEORY OF ELEMENTARY PARTICLES. II. ELECTRON THEORY OF SUPER-CONDUCTIVITY, by W. Heisenberg. 51 pages, 12 × 19 cm. New York, Cambridge University Press, 1949. No price.

The two lectures brought together in this slim book were delivered at the Cavendish Laboratory in December, 1947, as introductory discussions of two interesting, and as yet unresolved, topics in atomic theory. It is always a great pleasure to read Professor Heisenberg's lucid work, and he is perhaps at his best when, as in these lectures, he is directing his attention toward the understanding of physical fundamentals.

In the first lecture on the "Present Situation in the Theory of Elementary Particles" Heisenberg attempts to sketch, in broad outline, some necessary characteristics of a theory capable of describing the elementary particles. He suggests that the present quantum mechanical formalism based on the Correspondence Principle cannot be expected to apply for the relativistic energies at which the various elementary particles inter-convert. He doubts the existence of a primary Hamiltonian for the elementary particles and suggests that the various quantized field equations now employed for the description of particular particles will appear as low energy approximations in a complete theory. He finally describes briefly the S-matrix formalism which he has developed in several papers over the past few years as a possible alternative description of the elementary particles. At present, as Heisenberg admits, the S-matrix formalism is an "empty frame," but he implies that the attempt to discover the basic equations satisfied by the S-matrix is likely to prove more fruitful than the attempt to find a primary Hamiltonian.

In the second lecture Heisenberg describes his and H. Koppe's recent theory of superconductivity. In this theory Heisenberg assumes that at low temperatures the electrons at the top of the Fermi distribution may, through purely Coulomb interactions, form a condensed phase, which has the character of a low density electron lattice, through the superconductor. This assumption leads to the existence of current domains similar to the magnetic domains of a ferromagnet. Heisenberg describes the phenomena of super-conductivity in terms of the interactions between the Fermi electron gas, the ionic lattice, and the assumed electron lattice. He shows that the phenomenological equations of F. Landon may be derived from his assumptions.

Of course, all who are interested in the subjects discussed will be eager to read these lectures; Professor Heisenberg requires no commendation from a reviewer.

LEONARD EISENBUD

BOOK NOTES

EXPERIMENTAL GENERAL CHEMISTRY, by J. W. Neckers, T. W. Abbott and K. A. Van Lente. Second edition, 307 pages, diagrams, 20 × 27 cm. New York, Thomas Y. Crowell Company, 1949. Price, $2.25 (paper).

This laboratory manual for a course in general chemistry now appears in a revised edition to incorporate changes which observation of its use has indicated would make it more effective. About sixty per cent of the experiments have undergone at least partial revision. A feature which will commend itself both to the instructor and the student is a table correlating the experiments with seven different college chemistry texts.

ELEMENTARY MODERN PHYSICS, by Gordon F. Hull. 503 pages, illustrations, 14 × 22 cm. New York, Macmillan Company, 1949. Price, $5.25.

Originally published in 1936 under the title "An Elementary Survey of Modern Physics," this volume has been brought up to date. No marked change has been made in the treatment of the subject matter. Primarily a college textbook, it may interest some seeking a not too technical presentation of the ideas of modern physics since 1895.

ENCYCLOPEDIA OF CHEMICAL REACTIONS, compiled and edited by C. A. Jacobson. Volume III, 842 pages, 15 × 24 cm. New York, Reinhold Publishing Corp., 1949. Price, $12.00.

Beyond noting its appearance, there is little need to comment on this third volume of a useful reference work. Nineteen elements from cobalt to iridium, considering the elements alphabetically, are covered in this volume. This is more than in the previous volumes, since few reactions have been recorded in the chemical literature for the rarer elements, several of which are treated in this volume. Holmium and illumium are both represented by only one reaction; on the other hand copper has 610 and cobalt 822. The arrangement and indexes follow the standards set in the previous volumes.

AMERICAN COTTON HANDBOOK, by G. R. Merrill, A. R. Macormac and H. R. Mauersberger. Second revised edition, 943 pages, illustrations, 13 × 19 cm. New York, Textile Book Publishers, Inc., 1949. Price, $9.50.

Now appearing in a second revised edition, this standard work on the various phases of cotton manufacture, first published in 1941, has been brought up to date. The authors, the same as those of the first edition, have all had considerable experience in the textile field.

Several new chapters have been added, although to maintain the book within reasonable limits, some other material has been omitted. One of these new chapters is on cellulose and the cotton fiber, of particular interest in view of the newer developments in cotton finishing, many of which are based on fiber modification. The chapter on physical and chemical testing of fibers, yarns and fabrics has been split into two individual chapters and considerably augmented. Much new material has also been added on bleaching, dyeing and finishing. A chapter on "Nomenclature of Cotton Dyes" contains an alphabetical list of trade names giving the manufacturers and arranged by classes, according to fastness and dyeing properties. A comprehensive bibliography is included.

In addition to the major changes noted, the work shows evidence of many minor changes which result in clarification of the text. Containing much valuable information, this book should appeal to all those who are concerned with the manufacture of cotton goods.

CURRENT TOPICS

Ultra-Modern Shipping Center.—An ultra-modern $2,500,000 Shipping Center which employs many revolutionary new methods of material handling, and which will cut down from days to hours the time required to assemble and ship an order, has been opened at New Brunswick, N. J. by Johnson and Johnson.

The new Shipping Center, located just off New Jersey Highway 27 between Metuchen and New Brunswick, is set on a 58-acre plot, and its 207,000 gross square feet of floor space is sufficient to accommodate four regulation-size football fields.

This new unit is of modern industrial design. It brings into being an entirely new concept of warehousing, materials handling and shipping which is expected to revolutionize present methods in these fields.

An underground drag-line system pulls especially designed trucks through aisles along which some 500 different items are stored. An order assembler precedes these trucks, taking merchandise from piles of good stacked on pallets. An order calling for hundreds of different items can be assembled and shipped within a matter of hours, whereas previously as much as several days time were required.

To help keep this super-speed Shipping Center in high gear, three new inventions will be introduced for the first time:

A four wheel drag-line truck; a hand-operated label applicator; and a motor-driven dockboard are the new inventions designed to better the efficient operation of the new center. No employee will ever have to carry a carton more than four feet, or lift one more than four feet.

The drag-line trucks will follow the shipping employees around the center like pets anxious to carry boxes from the numerous vast stockpiles to the outgoing trucks, or railroad cars at the sidings. Each will carry an average of 900 lb., with a capacity of better than 4000 lb.

New hand-label applicators, twelve times faster than stenciling machines, will also help speed surgical dressings and other J & J products on to customers. They are light weight, hand-operated machines that apply labels, pre-printed with names and addresses of customers, on shipping cases while being loaded on a drag-line truck. Such on-the-spot labeling will minimize errors.

A mechanical dockboard for loading trucks is the third invention. Spanning the gap between building and carrier, the dockboard is electrically operated and can be swung into position in 10 seconds.

The new Shipping Center has two railroad tracks on a siding which leads to the main line of the Pennsylvania Railroad. Fifteen railroad "car spots," or loading and unloading bays, permit this number of cars to be loaded and unloaded simultaneously. Fourteen loading docks, seven for inbound and seven for outbound, will handle truck traffic. A special truck trailer parking lot will permit trailers to remain at the center until ready for loading. The Shipping Center's own truck tractor will haul the trailers to a loading bay and then back to the trailer parking lot loaded, where their own truck tractors

can conveniently remove them. This will eliminate idle time for the truck tractors.

As an order is received it is coded and shipping labels are prepared for insertion in an especially designed hand applicator which the order assembler uses. The assemblers have a sheet on which is listed the number of cartons or cases called for and the stock location numbers from which they are to be taken and placed on the small drag-line trucks. At the end of the dragline, trucks are removed from the line and the order is checked. If broken lots are required, they are assembled adjacent to this point from a series of shelves of loose merchandise. Here the small lots are placed on a belt-line, and wrapped. Then the packages are placed on marked shelves from which they are removed to the previously assembled whole-carton, or whole-case order shipment.

This new method of assembling and shipping an order will increase the capacity of the shipping plant ten times over that of the system previously in use.

The Shipping Center also employs an especially designed locator board at the receiving docks. It is a miniature of the reserve goods area and of its state of merchandise occupancy at any given time. This is supplemented by a series of visible index cards which serve as a perpetual inventory. As a pallet of merchandise is emptied in the order picking area a clip from the empty pallet is placed on the index card as a warning that additional reserve stock must be obtained from the factory.

Fruit "Essence" Now Free of Alcohol Tax.—Consumers should soon be getting food products with finer fruit flavors, say U. S. Department of Agricultural researchers. Changes in the alcohol-tax laws, ordered recently by Congress, now permit manufacture of fruit "essences" without payment of the $9-per-gallon tax formerly imposed. These flavor concentrates must still meet certain requirements, but Department scientists in the Bureau of Agricultural and Industrial Chemistry expect that the new regulations will result in rapid expansion of the fruit-essence industry—and in tastier fruit-flavored foods on your grocer's shelves.

A process for recovering and concentrating the volatile flavor and fragrance constituents of fresh apples and of grape juice has been developed by the Bureau at its Eastern Regional Research Laboratory in Philadelphia. Known technically as volatile fruit concentrates, the essences can be used by food manufacturers to enhance the taste and aroma of fruit products. A number of commercial companies are already set up to make concentrates from apples and other fruits by the Laboratory's method. Their demands for fresh fruit will help to expand markets for this year's tremendous apple crop.

The new alcohol-tax regulations governing production of volatile fruit concentrates were published in the *Federal Register* of September 27, 1949, and became effective on that date. They include revisions of the Internal Revenue Code in accordance with public Law 240, 81st Congress, which exempt fruit concentrates from the tax and various other provisions of the beverage-alcohol laws, provided certain requirements are met.

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